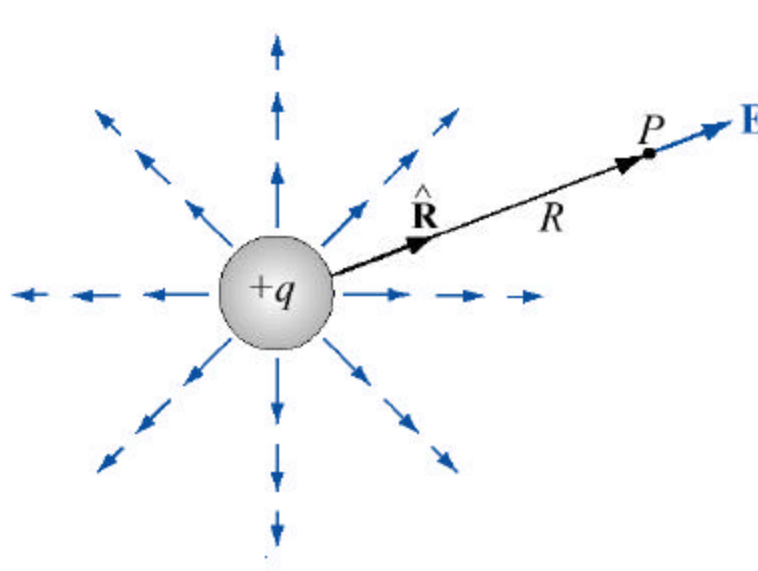


## LECTURE #2 Part B

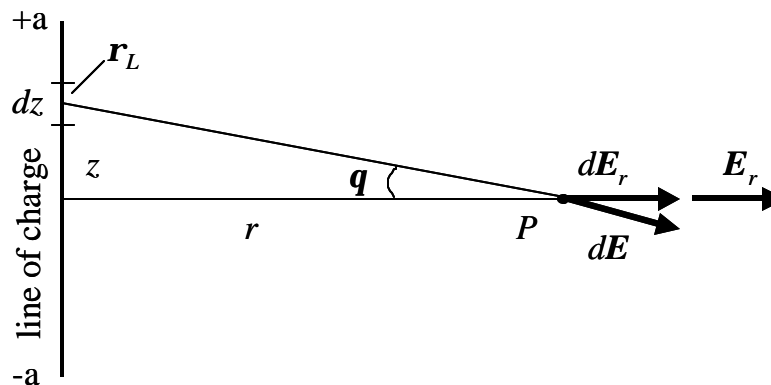


Electric Field Lines Due To A Point Charge  $Q$

When working with point charges, and the electric field or forces exerted on the charges, you may be required to perform vector analysis to define the unit vector  $\hat{\mathbf{r}}$  of the separation between the charges.

### LINE OF CHARGES

Shown below is a line of charge. The line has a length of  $2a$  and a linear charge density  $\rho_L$  [C/m]. The elemental length of charge is defined as  $dz$ . So what's the electric field at point  $P$  a distant  $r$  from the line?



Line Of Charge Distribution

The incremental electric field at point  $P$  from incremental line of charge  $dz$  is:

$$dE = \frac{\mathbf{r}_L}{4\mathbf{p}\mathbf{e}_o} \frac{dz}{(r^2 + z^2)^{3/2}} .$$

Note that the equation is simply a modified application of Coulomb's law where  $\rho_L$  is equivalent to  $Q / \text{length}$  and the radial distance between the elemental line of charge  $dz$  and point  $P$  is  $\sqrt{r^2 + z^2}$  .

The incremental electric field component perpendicular to the line of charge is then:

$$dE_r = dE \cos \mathbf{q} = dE \frac{r}{(r^2 + z^2)^{3/2}}$$

Now, since the point  $P$  that we are interested in is located such that one half (top) of the line of charge cancels the effects of the other half (bottom) of the line of charge, the components add to zero. The radial field at point  $P$  is then the integral of  $dE_r$  over the total length of charge  $2a$ :

$$E_r = \frac{\mathbf{r}_L r}{4\mathbf{p}\mathbf{e}_o} \int_{-a}^{+a} \frac{dz}{(r^2 + z^2)^{3/2}} = \frac{\mathbf{r}_L}{2\mathbf{p}\mathbf{e}_o r \sqrt{\left(\frac{r}{a}\right)^2 + 1}} \quad [\text{V/m}].$$

If the length of the line charge is much greater than the distance to the point  $P$ , then

$$E_r = \frac{\mathbf{r}_L}{2\mathbf{p}\mathbf{e}_o r} \quad [\text{V/m}].$$

### ELECTRIC POTENTIAL $V$ AND ITS GRADIENT $E$

Suppose we move a charge along a line from point  $a$  to point  $b$ . The *electric potential difference*  $\Delta V$  is then the work per unit charge in joules per coulomb. We EEs know this unit as *Volts*. So equation-wise, the electric potential difference is simply:

$$\Delta V = E \Delta x .$$

Or alternately, the electric field is  $\Delta E = \Delta V / \Delta x$  .

Alright, but there is some directionality associated with voltages so in terms of vectors,

$$\mathbf{E} = \hat{\mathbf{x}} E = -\hat{\mathbf{x}} \frac{\Delta V}{\Delta x} .$$

What the heck is the negative sign mean? Well, it reminds us that moving against an electric field means exerting positive work to move the point charge. Ok, so what is the voltage difference when moving a charge from point  $a$  to  $b$ ?

$$V = -\int_a^b \mathbf{E} \cdot d\mathbf{L} = -\int_a^b E \cos \mathbf{q} dL \quad [\text{V}].$$

So wait! We can generalize and say that

$$\mathbf{E} = -\left(\hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}\right)V = -\nabla V \quad [\text{V}].$$

So mathematically, we can call  $\mathbf{E}$  the **gradient of  $V$** .

The work required to transport a charge around a closed path is zero:

$$\oint \mathbf{E} \cdot d\mathbf{L} = 0.$$

Summarizing:

1. The line integral of the electric field  $\mathbf{E}$  is the potential  $V$  between two points.
2. The line integral of  $\mathbf{E}$  from infinity to a point gives the absolute potential of the point.
3. The gradient of  $V$  is the electric field  $\mathbf{E}$  at a *SINGLE POINT*.

### SUPERPOSITION OF POTENTIAL

The charges and its effects can be treated linearly. Therefore superposition principles apply:

$$V_P = \frac{1}{4\pi\epsilon_0} \sum_{n=1}^m \frac{Q_n}{r_n} \quad [\text{V}].$$

Just a few equations used to find electric potential for various distribution of charges are given below:

Line Charge: 
$$V_L = \frac{1}{4\pi\epsilon_0} \int \frac{\mathbf{r}_L}{r} dL \quad [\text{V}].$$

Surface Charge with surface charge density  $\rho_s$  and elemental surface  $ds$

$$V_S = \frac{1}{4\pi\epsilon_0} \iint \frac{\mathbf{r}_s}{r} ds \quad [\text{V}].$$

Volume Charge with volume charge density  $\rho_v$  and elemental volume  $dv$ :

$$V_v = \frac{1}{4\pi\epsilon_0} \iiint \frac{\mathbf{r}_v}{r} dv \quad [\text{V}].$$