Lecture #3/4

EQUIPOTENTIAL CONTOURS: ORTHOGONALITY

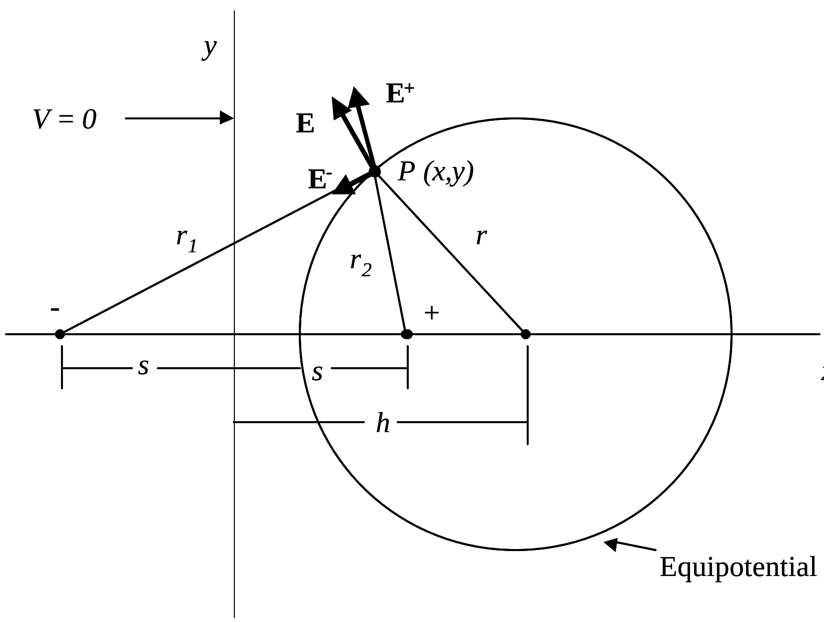
A field line indicates the direction of force on a positive test charge introduced to a field. So if a test charge is dropped in the field, it accelerates in the direction of the field line.

A uniform electric field, the **E** lines are parallel and the equipotential lines are perpendicular to the **E** direction and for a fixed voltage increments are uniformly spaced. The equipotentials are actually *planes* whose surfaces are perpendicular to **E**.

If we have a nonuniform field where for example, **E** is diverging, then the equipotential surfaces are curved and more widely spaced in the weaker field region.

MULTICONDUCTOR TRANSMISSION LINES

Transmission lines are used to transmit power, data, and other signals. A two wire transmission line, like what you might see on high voltage power lines or computer ribbon cables, consists of two long parallel wires spaced a distance 2*s* apart.



The potential at point *P* located at (*x*,*y*) is a scalar sum of the potentials due to the two lines. The potential difference between the positively charged line and *P* is

$$V^{+} = \frac{r_L}{2pe}\int_{r_2}^{s}\frac{dr}{r} = \frac{r_L}{2pe}\ln\frac{s}{r_2} \quad \text{[V]} .$$

For the negatively charged line

$$V^- = \frac{r_L}{2pe} \ln \frac{s}{r_1} \quad [\text{V}] .$$

The total voltage difference V between P and the chosen origin is

$$V = V^+ + V^- = \frac{r_L}{2pe} \ln \frac{r_1}{r_2} \quad [\text{V}].$$

The total electric field at P is the vector sum of the electric fields of the two wires

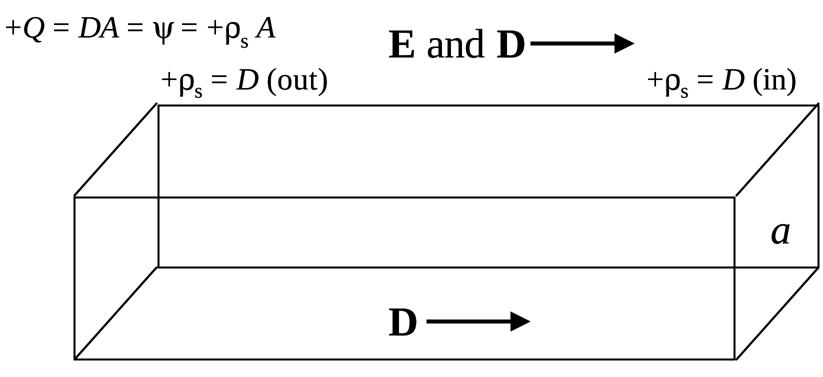
$$\mathbf{E} = \mathbf{E}^+ + \mathbf{E}^- = \frac{r_L}{2pe\, r_2} + \frac{r_L}{2pe\, r_1} \quad [\text{V/m}].$$

ELECTRIC FLUX AND ELECTRIC FLUX DENSITY - GAUSS' LAW

The electric flux y is the surface integral of the normal component of **E** over the area:

$$y = \iint e\,\mathbf{E} \bullet d\mathbf{s} \quad .$$

Suppose we have a uniform field E that we can "box in" as shown below:



Flux “tube” with $\psi = Da$ for any cross section

The flux over the small area a is $y = eEa$ [C] and the total flux between the two ends of the flux tube is $y = eEA$ [C] .

The *flux density* D is defined as charge per unit area: $D = \frac{y}{A} = eE$ [C m^{-2}].

In vector form is: $\mathbf{D} = e\mathbf{E}$ [C m^{-2}].

The flux density is also called the *electric displacement*.

NON-UNIFORM SURFACE CHARGE

The total flux over a spherical surface is:

$$y = \oint\!\!\oint_s \mathbf{D} \bullet d\mathbf{s} = Q$$

The electric field from a point charge is equal to $Q/4\pi\varepsilon r^2$. Since $\mathbf{D} = e\mathbf{E}$ and $d\Omega = \sin q \, dq \, df$:

$$y = \frac{Q}{4p}\iint d\Omega = \frac{Q}{4p}\int_0^{2p}\int_0^{p} sin q \, dq \, df$$
$$= \frac{Q}{4p}\left[-cos q\right]_0^{p}\int_0^{2p} df = \frac{Q}{4p}\bullet 2\bullet 2p = Q$$

VOLUME CHARGE AND GAUSS' LAW

$$\oint_s \mathbf{D} \bullet d\mathbf{s} = \oint_v r \, dv = Q \quad \text{Gauss' Law}$$

DIVERGENCE

$$\oint\!\!\oint \mathbf{D} \bullet d\mathbf{s} = \iiint r \, dv = Q$$

and the divergence of **D** is:

$$\nabla \bullet \mathbf{D} = r$$

where $\nabla \bullet D = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = r \quad [\text{Cm}^{-3}]$.

LAPLACE'S AND POISSON'S EQUATIONS

$$\nabla^2 V = -\frac{r}{e} \quad \text{Poisson's Equation}$$

and in free space, Poisson's Equation simplifies to:

$$\nabla^2 V = 0 \quad \text{Laplace's Equation .}$$

BOUNDARY CONDITIONS

Things to remember:

The tangential electric field **E** is continuous across the boundary surface.

$$E_{t1} = E_{t2} \text{ Tangential } E.$$

The discontinuity in the tangential magnetic field **H** is equal to the surface current $\mathbf{J}_s$.

$$\hat{n}\times\left(\vec{\mathbf{E}}_1 - \vec{\mathbf{E}}_2\right) = \mathbf{0}$$

$$\hat{n}\times\left(\vec{\mathbf{H}}_1 - \vec{\mathbf{H}}_2\right) = \vec{\mathbf{J}}_s \quad .$$

1. For two media having finite conductivities, both tangential electric field **E** and magnetic field **H** are continuous across the boundary.
2. On the surface of a perfect conductor, the tangential field **E** is zero, and the surface current $\vec{\mathbf{J}}_s = \hat{n}\times\vec{\mathbf{H}}$, where $\hat{n}$ is the unit vector normal to the conductor's surface. That is, no electromagnetic field exists in a perfect conductor.

The normal component of **B** is continuous across the boundary surface. The discontinuity in the normal component of **D** is equal to the surface charge density r_s.

$$B_{1n} - B_{2n} = 0 \quad \text{or} \quad \vec{\nabla}\bullet\vec{\mathbf{B}} = 0$$

$$D_{1n} - D_{2n} = r_s \quad \text{or} \quad \vec{\nabla}\bullet\vec{\mathbf{D}} = r_s \quad .$$

<u>CAPACITORS AND CAPACITANCES</u>

$$C = \frac{Q}{V} \quad \text{[C/m] or [F]}$$

$$Q = r_s A = DA = eEA \quad \text{[C]}$$

A = plate area in m^2
r_s = surface charge density in C/ m^2
D = electric flux density in C/ m^2
E = electric field in V/ m

$$C = \frac{Q}{V} = \frac{DA}{Ed} = \frac{eA}{d}$$

The capacitance per unit length of a capacitor cell equals the permittivity of the medium.

CAPACITANCE ENGERGY AND ENERGY DENSITY

Total energy stored by a capacitor is:

$$W = \frac{1}{C}\int_0^Q q\,dq = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

But $V=Eh$ and $Q = eEA$ where h is the space between the capacitor plates and A is the area of the plates. So,

$$W = \frac{1}{2}QV = \frac{1}{2}eEAEh = \frac{1}{2}eE^2Ah$$

When we divide the work by the volume of the capacitor, we obtain the energy density:

$$w = \frac{1}{2}eE^2 \quad .$$