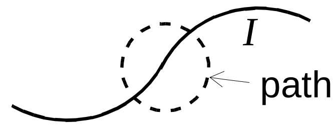
Ampere’s Law

$$\oint \vec{H} \cdot d\vec{l} = I$$

Biot and Savart's Law is equivalent to **Ampere's Law**, (Chapter 5.4 - 2) which states that:

$$\oint \vec{H} \cdot d\vec{l} = I$$



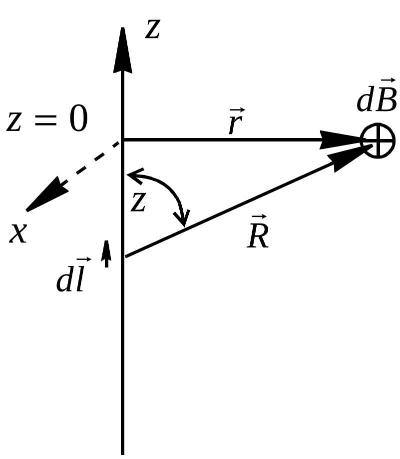
"$\oint$" refers to any path, and I is the total current flowing *through* the area enclosed by the path. That is, the current must pierce the area within the path! I is an algebraic sum. For currents in the opposite direction, $+$ and $-$ signs indicate directions.

Right Hand Rule

Positive direction for the current is the thumb, if fingers of the right hand point in the direction of the path.

Demonstration of Ampere's Law

First, let us calculate the field of an infinitely long, straight current. Then apply Ampere's Law.



Integrate over $\theta,\ 0 \rightarrow \pi$

Recall Law of Biot and Savart

$$d\vec{B} = \frac{\mu_o}{R^2\, 4\pi}\, I\, d\vec{l} \times \hat{e}_R$$

$$d\vec{l} \times \hat{e}_R = \hat{e}_\phi \sin\theta\, d\vec{l} = \hat{e}_\phi \sin\theta\, dz$$

$$R^2 = \left(\frac{r}{\sin\theta}\right)^2$$

$z = -\,r\cot\theta$, or $dz = r\csc^2\theta\, d\theta$,

or $dz = \dfrac{r\, d\theta}{\sin^2\theta}$

Putting together: $d\vec{B} = \dfrac{I\,\mu_o}{4\pi}\left(\dfrac{\sin\theta}{r}\right)^2 \hat{e}_\phi \sin\theta\, \dfrac{r\, d\theta}{\sin^2\theta}$

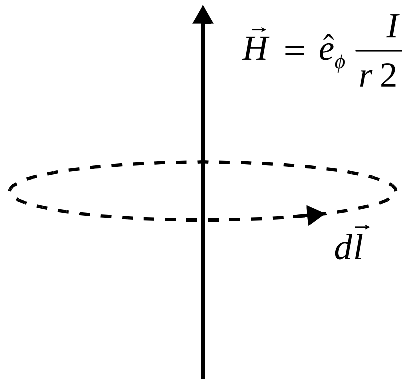
$$\vec{B} = \hat{e}_\phi \int_0^\pi \frac{I\,\mu_o}{4\pi}\frac{\sin\theta}{r}\, d\theta = \hat{e}_\phi \frac{I\,\mu_o}{r\,4\pi}\left[-\cos\theta\right]_0^\pi$$

$$\boxed{\vec{B} = \hat{e}_\phi \frac{I\,\mu_o}{r\,2\pi} \qquad \vec{H} = \hat{e}_\phi \frac{I}{r\,2\pi}}$$

Note that the magnetic field intensity $\vec{H}$ has units of amp/m, the same as the surface current.

Demonstration of Ampere's Law (continued)

We just have shown directly from the Law of Biot and Savart that the field of an infinite straight wire carrying I in the z direction is:



$$\vec{H} = \hat{e}_\phi \frac{I}{r\,2\pi}$$

Verify Ampere's Law!

Choose a path of integration along consant *r*.

Then $d\vec{l} = r\,d\phi\,\hat{e}_\phi$

and $\oint \vec{H}\cdot d\vec{l} = \int_0^{2\pi} H(r)\,r\,d\phi$

so $\int \vec{H}\cdot d\vec{l} = \int_o^{2\pi} \frac{I}{2\pi}\frac{r}{r}\,d\phi = I$

Using Ampere's Law to solve problems

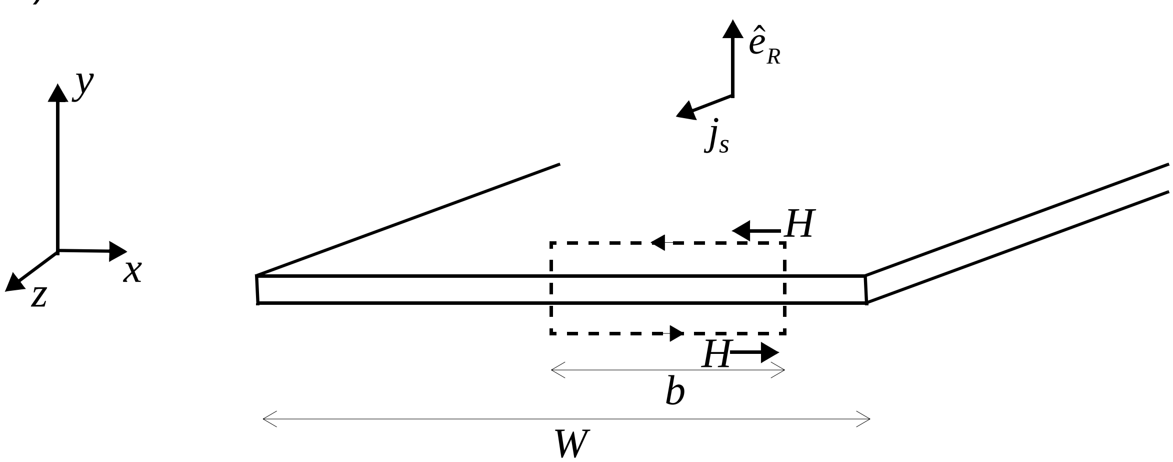
If we have a symmetric problem, such that the direction of $\vec{H}$ is known and is constant over a chosen path, then:

$$\oint \vec{H}\cdot d\vec{l} = H\text{ (path length)} = I$$

Therefore: $H = I$ / pathlength

Example:

Consider an infinite current sheet with current density j_s.
(Practical case: W is width, but can be finite to a reasonable approximation.)



Let the current density be j_s, such that $I = j_s W$ (sheet current density).
Now consider symmetry.
H will not vary with x (infinite sheet).

Since we take j_s to be directed along $\hat{e}_z$, H will be directed along $\hat{e}_x$.

In particular:

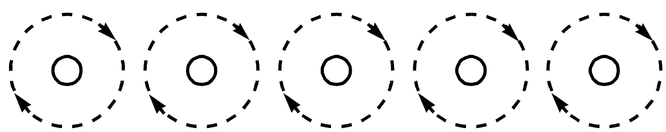
$$\vec{H} = -H\hat{e}_x \quad \text{for } y > 0$$
$$= H\hat{e}_x \quad \text{for } y < 0$$

Then Ampere's Law says:

$$-b(-H) + bH = j_s b = \frac{Ib}{W}$$

Therefore: $H = \frac{j_s}{2}$, or $\frac{I}{2W}$.

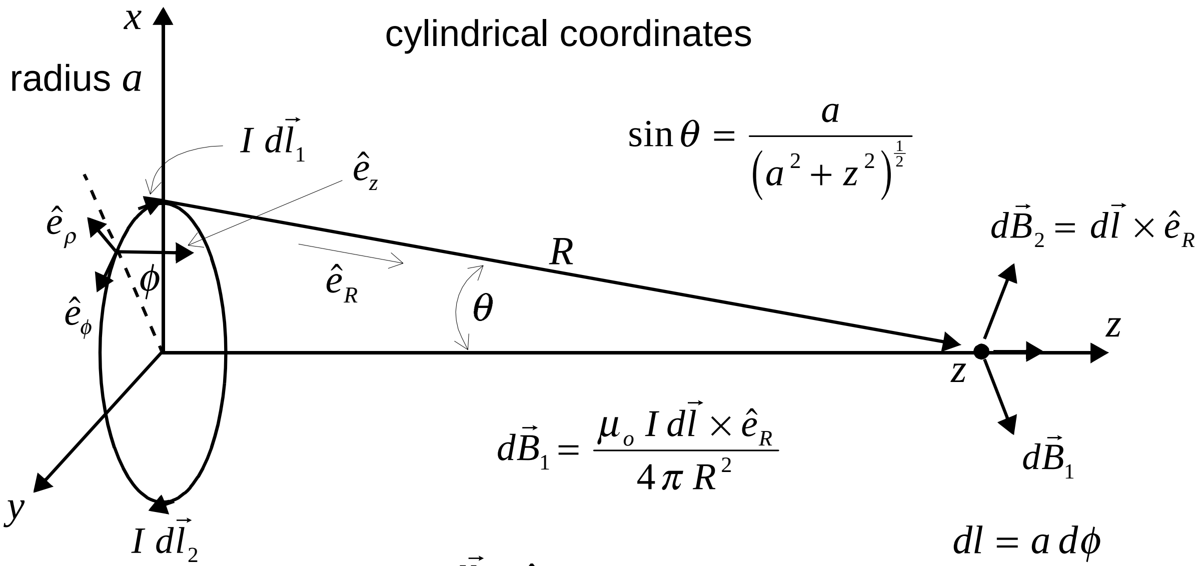
Discussion of why H is as shown:



Vertical components cancel, and horizontal components add.

Example:

Use Law of Biot and Savart to calculate the magnetic field on the axis of symmetry of a single current loop.



$$d\vec{B} = \frac{\mu_o\, I\, d\vec{l}\times\hat{e}_R}{4\pi\left(a^2+z^2\right)}$$

Consider $I\,d\vec{l}_2$ to be 180° out of phase.

$$dB_z = dB_1 \sin\theta = \frac{\mu_o\, I\, a\, d\phi \sin\theta}{4\pi\left(a^2+z^2\right)}$$

$$= \frac{\mu_o\, I\, a\, d\phi}{4\pi\left(a^2+z^2\right)}\,\frac{a}{\left(a^2+z^2\right)^{\frac{1}{2}}}$$

$$= \frac{\mu_o\, I\, a^2 d\phi}{4\pi\left(a^2+z^2\right)^{\frac{3}{2}}} \Rightarrow B_z = \frac{\mu_o\, I\, a^2\, 2\pi}{4\pi\left(a^2+z^2\right)^{\frac{3}{2}}}$$

$$\vec{B} = \frac{\mu_o\, I\, a^2\, \hat{e}_z}{2\left(a^2+z^2\right)^{\frac{3}{2}}}$$