## Antenna Array Notes

For a dipole array antenna with the arrays lined up on the $y$-axis, the following field pattern characteristics are valid. The array pattern factor is:

$$
|F(\theta, \phi)|_{y-a x i s}=\frac{1}{N}\left|\frac{\sin \left(N \frac{k d \sin \theta \sin \phi+\psi}{2}\right)}{\sin \left(\frac{k d \sin \theta \sin \phi+\psi}{2}\right)}\right| .
$$

For arrays lined up on x -axis, the array pattern factor is:

$$
|F(\theta, \phi)|_{x-a x i s}=\frac{1}{N}\left|\frac{\sin \left(N \frac{k d \sin \theta \cos \phi+\psi}{2}\right)}{\sin \left(\frac{k d \sin \theta \cos \phi+\psi}{2}\right)}\right| .
$$

## 1. Main-beam direction

The maximum value of the array pattern factor, $|F(\theta, \phi)|_{\text {y-axis }}$, occurs when $k d \sin \theta \sin \phi+\psi=0$, which leads to

$$
\sin \theta \sin \phi=-\frac{\psi}{k d}
$$

Two special cases are of particular importance.
(a) Broadside Array

For a broadside $y$-axis array, maximum radiation occurs at a direction perpendicular to the line of the array: that is, $\phi=0$. This requires that the antennas are excited in phase, or $\psi=0$.
(b) Endfire Array

For an endfire $y$-axis array, maximum radiation occurs at $\phi= \pm \frac{\pi}{2}$. The phase difference between antennas is: $\psi=-k d \sin \phi \sin \theta=-k d$.

## 2. Sidelobe locations

Sidelobes are minor maxima that occur approximately when the numerator of the $y$-axis pattern function is a maximum: that is, when $\left|\sin \left(N \frac{k d \sin \theta \sin \phi+\psi}{2}\right)\right|=1$, or when

$$
N \frac{k d \sin \theta \sin \phi+\psi}{2}= \pm(2 m+1) \frac{\pi}{2}, \quad m=1,2,3, \ldots
$$

The first sidelobes occur when

$$
N \frac{k d \sin \theta \sin \phi+\psi}{2}= \pm \frac{3 \pi}{2} \quad(\text { for } m=1)
$$

Note that $N \frac{k d \sin \theta \sin \phi+\psi}{2}= \pm \frac{\pi}{2}$ (for $m=0$ ) does not represent locations of sidelobes because they are still within the main-beam region.
3. First sidelobe level

An important characteristic of the radiation pattern of a $y$-axis array antenna is the level of the first sidelobe compared to that of the main beam, since the former is usually the highest of all sidelobes. All sidelobes should be kept as low as possible in order that most of the radiated power be concentrated in the main-beam direction and not be diverted to sidelobe regions. The amplitude of the first sidelobe is,

$$
\frac{1}{N}\left|\frac{1}{\sin \left(\frac{3 \pi}{2 N}\right)}\right|
$$

