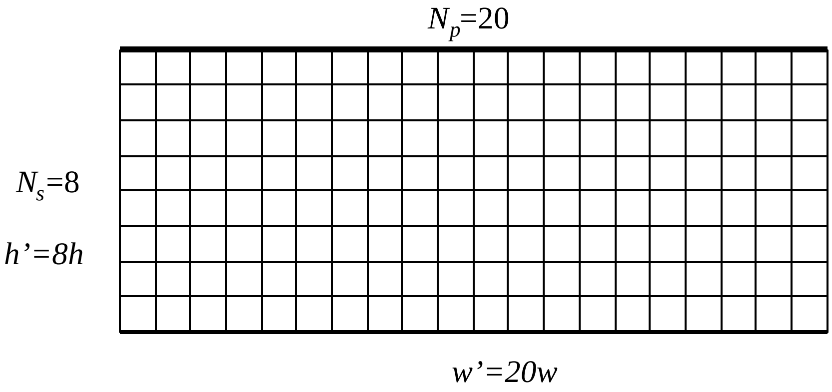
LECTURE #4

TWIN-STIP AND MICROSTRIP TRANSMISSION LINES

You can estimate the capacitance per unit length by defining and drawing equipotential squares between the capacitor plates:

$$\frac{C}{l} = e\frac{\text{Stripwidth}}{\text{Stripspacing}} = e\frac{N_p}{N_s}$$



Field Map Of Twin Strip Transmission Line

If a voltage of 16 V is applied between the plates and the individual equipotential cells are 8mm per side, the electric field between the plates is:

$$E = \frac{V}{\text{spacing}} = \frac{16}{0.008} = 2000 \ \text{[V/m]}$$

ELECTRIC CURRENTS

Force on a test charge in an electric field: $\mathbf{F} = e\mathbf{E}$ [N] .

Acceleration of that test charge is: $\mathbf{a} = \mathbf{F}/m$ [ms^{-2}] where m is the mass of the charge in kg.

In free space, the charged particle will accelerate indefinitely with a constant **E**. But in a real media, particles collide losing energy and radiating energy. In essence this action restrains the charged particles to a constant average velocity called the *drift velocity* $\mathbf{v}_d$. $\mathbf{v}_d$ has the same direction as **E** and related to a constant called the mobility μ_m.

The ***drift velocity*** is: $\mathbf{v}_d = \mu_m \mathbf{E}$ [m/s] where μ_m has units of [m^2 V^{-1} s^{-1}] . High mobility implies good electrical conductors.

Suppose that a medium has a cross sectional area A and contains many free moving charged particles of volume density r. When these charged particles move, they form a current:

$$\mathbf{I} = \mathbf{v_d}\, rA \ \ [\mathrm{A}] \text{ or } [\mathrm{C/s}] \ .$$ ***Current***

OHM's LAW

The potential difference or voltage between the ends of a conductor is equal to the product of its resistance and the current:

$$V = I\,R\,.$$ ***Ohm's Law***

The current I is equal to the current density J in [A/m^2] multiplied by the cross sectional area of the material: $I = JA$.

But the resistance and voltage are defined as

$$R = \frac{w}{s\ A} \quad \text{and } V = Ew\ ,$$

where A is the cross sectional area of the material perpendicular to the current flow, s is the conductivity in [S/m], and w is the length of the material in the direction of current flow.

These relationships yield:

$J = s\ E$ ***Ohm's Law At A Point In A Uniform Density Material*** .

We know that power is: $P = I^2 R$. the energy consumed by a device in a time T is defined by Joule's Law:

$$W = PT = I^2\,RT\,. \ \ [\mathrm{J}] \text{ or } [\mathrm{W\text{-}s}]$$

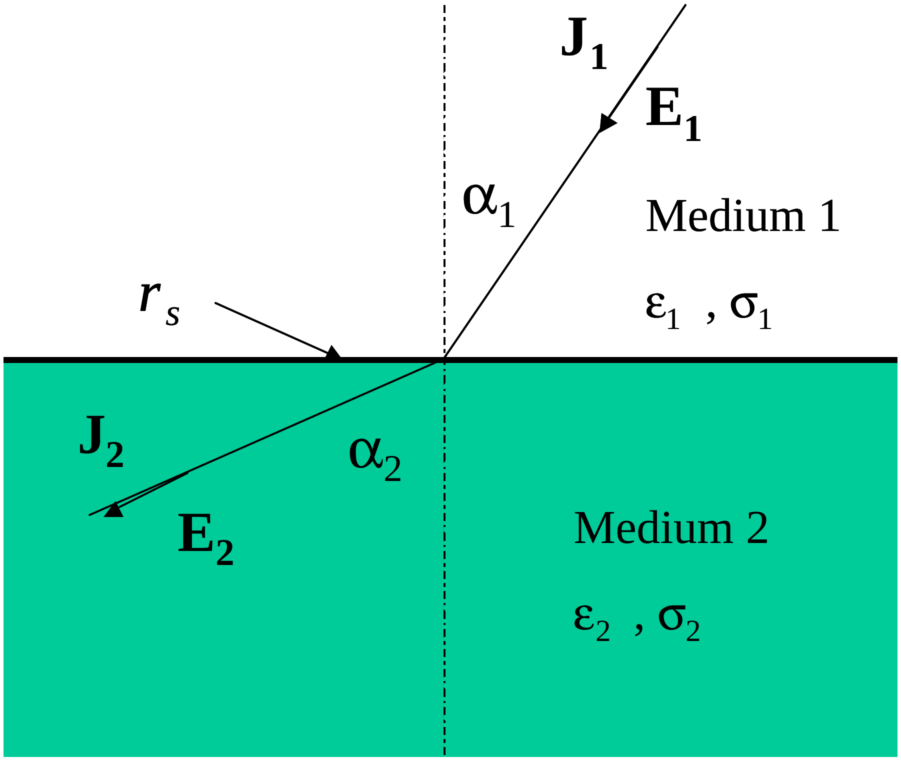
Kirchhoff's Current Law states that:

$$\tilde{N} \bullet \mathbf{J} = 0\ .$$

BOUNDARY CONDITIONS OF CONDUCTING MEDIA

The normal components of the current density at the boundary between two conducting media are continuous:

$$J_{n1} = J_{n2}\ .$$



Boundary Condition Between Two Conducting Media

Also,

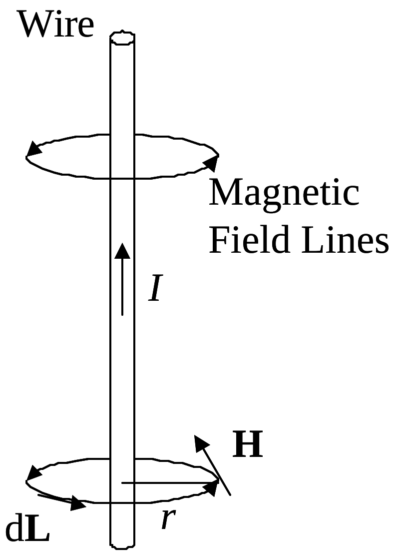
$$E_{t1} = E_{t2} \qquad \text{or} \qquad J_{t1}/s_1 = J_{t2}/s_2$$

The angle relationship is: $$\frac{\tan a_1}{\tan a_2} = \frac{s_1}{s_2} \quad .$$

MAGNETIC FIELDS OF ELECTRIC CURRENTS

A wire with current *I* is surrounded by a magnetic field as defined by the ***Biot-Savart Law***:

$$d\mathbf{H} = \frac{Id\mathbf{L}\sin q}{4p\, r^2} \quad [\text{A/m}] \ .$$



Magnetic Field **H** Around A Current-Carrying Wire

An alternate equation for the ***Biot-Savart Law*** for differential magnetic field $d\mathbf{H}$ generated by a steady-state current I flowing through a differential length $d\mathbf{L}$ is:

$$d\mathbf{H} = \frac{I}{4\pi}\frac{d\mathbf{L}\times\hat{\mathbf{R}}}{R^2} \quad \text{[A/m]}$$

where $\mathbf{R} = r\hat{\mathbf{R}}$ is the distance vector between $d\mathbf{L}$ and the observation point. It is important that the direction of the magnetic field is defined such that $d\mathbf{L}$ is along the direction of the current I and the unit vector $\hat{\mathbf{R}}$ points from the current element to the observation point. The differential magnetic field varies with R^{-2}. Note that **H** is orthogonal to the plane containing the direction of the current element $d\mathbf{L}$ and the distance vector **R**.

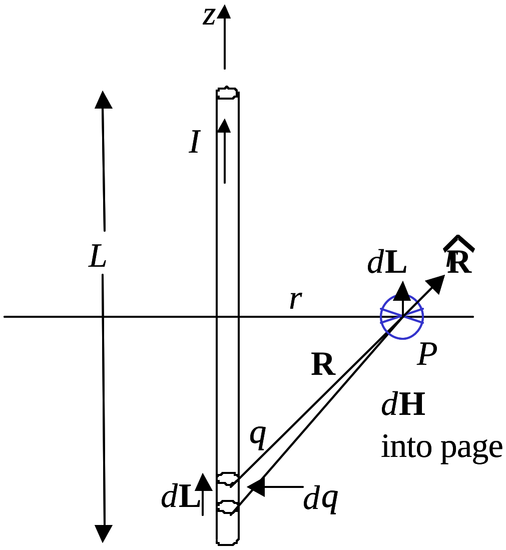
To determine the Total Magnetic Field **H** due to a conductor of finite size, we need to sum up the contributions due to all the current elements making up the conductor. Hence, the Biot-Savart Law becomes

$$\mathbf{H} = \frac{I}{4\pi}\int_L \frac{d\hat{\mathbf{L}}\times\hat{\mathbf{R}}}{R^2} \quad \text{[A/m]},$$

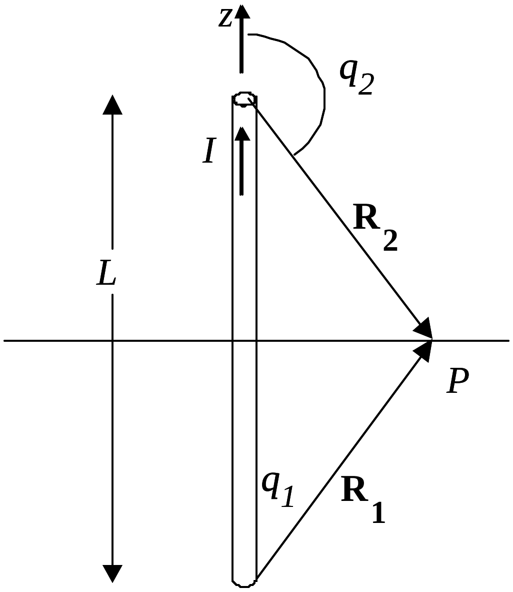
where L is the line path along which I exists.

Example: Magnetic field of a Linear Conductor

In the figures below, determine the magnetic field at point P located at distance r in the x-y plane in free space.



Linear Conductor Length *L* with Current *I*



Limiting Angles Each Measured Between Vector *I d***L** And The Vector Connecting The End Of The Conductor Associated With That Angle To Point *P*

Form the figure, the current element $d\mathbf{L} = \hat{\mathbf{z}}dz$ and $d\mathbf{L} \times \widehat{\mathbf{R}} = dz\left(\hat{\mathbf{z}} \times \widehat{\mathbf{R}}\right) = \widehat{\phi}\,\sin\theta\,dz$. Apply the Biot-Savart law:

$$\mathbf{H} = \frac{I}{4\pi}\int_{z=-L/2}^{z=L/2} \frac{d\widehat{\mathbf{L}} \times \widehat{\mathbf{R}}}{R^2} = \hat{\phi}\,\frac{I}{4\pi}\int_{-L/2}^{L/2} \frac{\sin\theta}{R^2}\,dz$$

For convenience, we will convert the integration variable from z to q using the transformations:

$$R = r \csc q$$
$$z = -r \cot q$$
$$dz = r \csc 2q\, dq$$

Inserting the transformations into the integral form of the Biot-Savart law yields

$$\begin{aligned}\mathbf{H} &= \hat{f}\frac{I}{4p}\int_{q_1}^{q_2}\frac{\sin q \csc^2 q\, dq}{r^2 \csc^2 q} \\ &= \hat{f}\frac{I}{4p}\int_{q_1}^{q_2}\sin q\, dq \\ &= \hat{f}\frac{I}{4p}\left(\cos q_1 - \cos q_2\right)\end{aligned}$$

From the right triangle in the second figure of the example,

$$\cos q_1 = \frac{L/2}{\sqrt{r^2 + (L/2)^2}} \quad \text{and} \quad \cos q_2 = -\cos q_1 = \frac{-L/2}{\sqrt{r^2 + (L/2)^2}} .$$

Therefore, $\mathbf{H} = \hat{f}\dfrac{IL}{2p\, r\sqrt{4r^2 + L^2}}$ [A/m].

For an infinitely long wire such that $L >> r$, the magnetic field expression reduces to:

$$\mathbf{H} = \hat{f}\frac{I}{2p\, r} \text{ [A/m].}$$