## THE SMITH CHART - WHAT IS IT?

It is a polar plot of the complex reflection coefficient (called $r$ herein), or also known as the 1-port scattering parameter $s$ or $s_{11}$, for reflections from a normalized complex load impedance $z = r + jx$; the normalized impedance is a complex dimensionless quantity obtained by dividing the actual load impedance $Z_L$ in ohms by the characteristic impedance $Z_o$ (also in ohms, and a real quantity for a lossless line) of the transmission line.

The contours of $z = r + jx$ (dimensionless) are plotted on top of this polar reflection coefficient (complex $r$) and form two orthogonal sets of intersecting circles. The center of the SMITH chart is at $r = 0$ which is where the transmission line is "matched", and where the normalized load impedance $z=1+j0$; that is, the resistive part of the load impedance equals the transmission line impedance, and the reactive part of the load impedance is zero.

The complex variable $z = r + jx$ is related to the complex variable gamma by the formula

$$z = r + jx = \frac{1 + r}{1 - r}$$

and of course, the inverse of this relationship is

$$r = \frac{z - 1}{z + 1} = \frac{(r-1) + jx}{(r+1) + jx}$$

From this chart we can read off the value of gamma for a given $z$, or the value of $z$ for a given gamma. The modulus of gamma, which is written $|r|$, is the distance out from the center of the chart, and the phase angle of gamma, written $\arg(r)$, is the angle around the chart from the positive $x$ axis. There is an angle scale at the perimeter of the chart.

On a lossless transmission line the waves propagate along the line without change of amplitude. Thus the size of gamma, or the modulus of $r$, $|r|$, doesn't depend on the position along the line. Thus the impedance "transforms" as we move along the line by starting from the load impedance $z = Z_L/Z_o$ and plotting a circle of constant radius $|r|$ travelling towards the generator. The scale on the perimeter of the SMITH chart has major divisions of 1/100 of a wavelength; by this means we can find the input impedance of the loaded transmission line if we know its length in terms of the wavelength of waves travelling along it.

The plot you usually see is the inside of the region bounded by the circle $|r| = 1$. Outside this region there is reflection gain; in this outside region, the reflected signal is larger than the incident signal and this can only happen for $r < 0$ (negative values of the real part of the load impedance). Thus the perimeter of the SMITH chart as usually plotted is the $r = 0$ circle, which is coincident with the $|r| = 1$ circle. The $r = 1$ circle passes through the center of the SMITH chart. The point $r = 1$ angle 0 is a singular point at which $r$ and $x$ are multi-valued.

The SMITH chart represents both impedance and admittance plots. To use it as an admittance plot, turn it through 180 degrees about the center point. The directions "towards the generator" and "towards the load" remain in the same sense. The contours of constant resistance and constant reactance are now to be interpreted as constant (normalized) conductance $g$, and (normalized) susceptance s respectively.

To see this property of the SMITH chart we note first that the admittance *y* is the reciprocal of the impedance *z* (both being normalized). Thus inverting the equation above we see that

$$y = g + js = \frac{1 - r}{1 + r}$$

and this is the same formula that we had above if we make the substitution $r$ --> $(-r)$. Of course, inverting the SMITH chart is the same as rotating it though 180 degrees or π radians, since

$$-r = r\, e^{jp}.$$

Admittance plots are useful for shunt connected elements; that is, for elements in parallel with the line and the load.

**Why is one circuit of the SMITH chart only half a wavelength?**

We remember that the SMITH chart is a polar plot of the complex reflection coefficient, which represents the ratio of the complex amplitudes of the backward and forward waves.

Imagine the forward wave going past you to a load or reflector, then travelling back again to you as a reflected wave. The total phase shift in going there and coming back is twice the phase shift in just going there. Therefore, there is a full 360 degrees or 2π radians of phase shift for reflections from a load HALF a wavelength away. If you now move the reference plane a further HALF wavelength away from the load, there is an additional 360 degrees or 2π radians of phase shift, representing a further complete circuit of the complex reflection (SMITH) chart. Thus for a load a whole wavelength away there is a phase shift of 720 degrees or 4π radians, as the round trip is 2 whole wavelengths. Thus in moving back ONE whole wavelength from the load, the round trip distance is actually increasing by TWO whole wavelengths, so the SMITH chart is circumnavigated twice.

**A note on the precision of the SMITH chart**

It might be thought that the SMITH chart is only a rough and ready calculator since points can only be determined and plotted on it to within a certain tolerance depending on the size of the print copy of the chart. However, the angular scale at the edge has divisions of 1/500 of a wavelength (0.72 degrees) and the reflection coefficient scale can be read to a precision of 0.02. A little thought shows that this is quite sufficient for most purposes. For example, if the wavelength in coaxial cable at 1 GHz is 20 cm, the SMITH chart locates the position along the cable to 20/500 cm or 0.4 mm and it is clear to anyone who has handled cable at 1 GHz that it cannot be cut to this precision.

Should more precision be required, an enlarged section of the chart can easily be made with most photocopy machines. A corollary of these remarks about precision is that many students over-specify the accuracy of their answers to transmission line problems. Normally 3 significant figures in the reflection coefficient is more than ample; angles can be quoted to the nearest degree and normalized impedances and admittances to about 1%. For, it is going to be very difficult to construct a real circuit which is accurately described by more precision than this.

Since many people now rely on computer modeling of transmission lines, they have lost sight of the precision limits of the descriptions of their physical circuit implementations. If your matching circuit requires parameters to be chosen more closely than about a percent in order to work, you probably won't be able to make it physically.