Microwave Filters

Passbands and Stopbands in Periodic Structures

Periodic structures generally exhibit passband and stopband characteristics in various bands of wave number determined by the nature of the structure. This was originally studied in the case of waves in crystalline lattice structures, but the results are more general. The presence or absence of propagating wave can be determined by inspection of the k-β or ω-β diagram. For our purposes it's enough to know the generality that periodic structures give rise to bands that are passed and bands that are stopped.

To construct specific filters, we'd like to be able to relate the desired frequency characteristics to the parameters of the filter structure. The general synthesis of filters proceeds from tabulated low-pass prototypes. Ideally, we can relate the distributed parameters to the corresponding parameters of lumped element prototypes.

As we will see, the various forms of filter passband and stopband can be realized in distributed filters as well as in lumped element filters.

Over the years, lumped element filters have been developed that are non-minimum phase; that is, the phase characteristics are not uniquely determined by the amplitude characteristics. This technique permits the design of filters for communications systems that could not be constructed using only minimum phase filter concepts. This generally requires coupling between multiple sections, and can be extended to distributed filters.

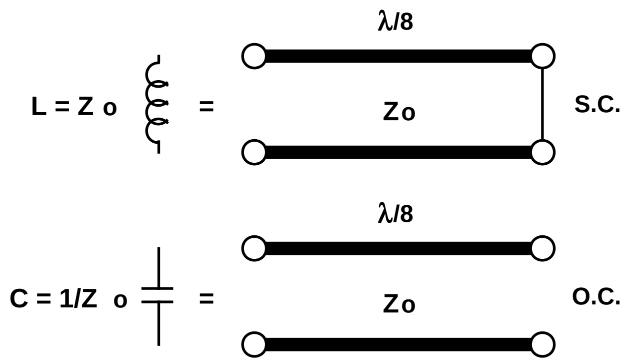
The design of microwave filters is comprehensively detailed in the famous Stanford Research Institute publication[1] *Microwave Filters, Impedance Matching Networks and Coupling Structures*.

Richard's Transformation and Kuroda's Identities (λ/8 Lines)

Richard's Transformation and Kuroda's Identities focus on uses of λ/8 lines, for which $X = jZ_o$. Richard's idea is to use variable Z_o (width of microstrip, for example) to create lumped elements from transmission lines. A lumped low-pass prototype filter can be implemented using λ/8 lines of appropriate Z_o to replace lumped L and C elements.

So if we need an inductance of L for a prototype filter normalized to cutoff frequency $\omega_c = 1$ and admittance $g_o = 1$, we can substitute a λ/8 transmission line stub that has $Z_o = L$. The last step of the filter design will be to scale the design to the desired ω_c and Z_o (typically 50Ω).

[1] Matthaei, Young & Jones, *Microwave Filters, Impedance Matching Networks and Coupling Structures*, McGraw Hill, 1965, now available from Artech House, 1980



The $\lambda/8$ transmission line sections are called commensurate lines, since they are all the same length in a given filter.

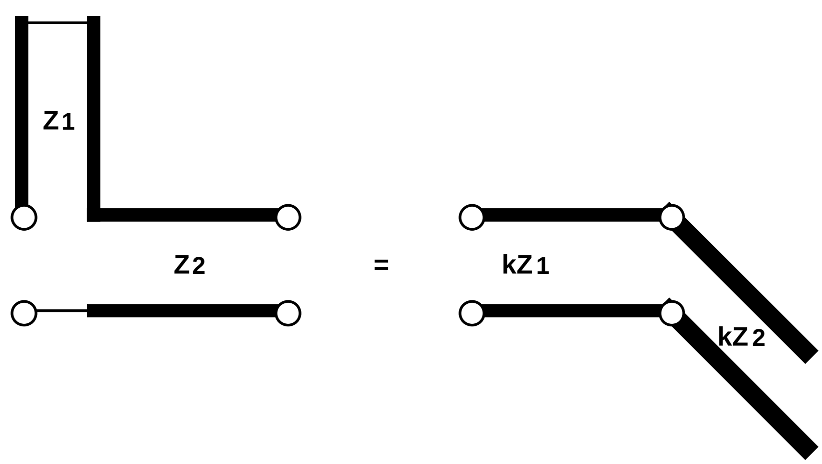
Kuroda's idea is use the $\lambda/8$ line of appropriate Z_o to transform awkward or unrealizable elements to those with more tractable values and geometry. As an example, the series inductive stub in the diagram here can be replaced by a shunt capacitive stub on the other end of the $\lambda/8$ line, with different values of characteristic impedance determined by

$$k = n^2 = 1 + \frac{Z_1}{Z_2}$$

As an example, consider a prototype network with the values

$$L = Z_1 = 0.5 \text{ and } Z_2 = 1 \quad k = n^2 = 1 + \frac{Z_1}{Z_2} = 1.5$$

So for the equivalent network, the series transmission line element has $Z = 1.5Z_1 = 0.75$ and the shunt capacitive stub has $Z = 1.5Z_2 = 1.5$. Kuroda's four identities are a means of eliminating series stubs that arise from series L or C in prototype networks.



This diagram shows the equivalence for the most commonly used identity, which removes a series stub by transforming it to a shunt stub along with adjustment of characteristic impedances of the $\lambda/8$ lines.

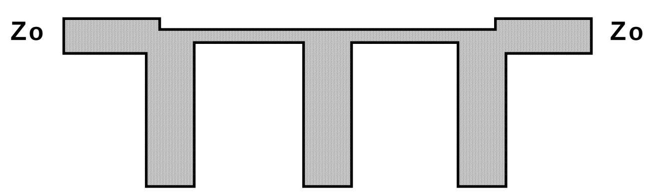
Low Pass Filter Using Stubs

The prototype lowpass LC structure employs series inductors, so a direct conversion to transmission line stubs by Richard's transformation would result in series stubs. However, we can use the Kuroda identity for series inductors to create a structure that has only series transmission line sections and shunt open stubs.

In order to do this we must be aware that we should begin by adding unit elements ($\lambda/8$ transmission lines of $Z_o = 1$) at each end of the filter, so that there will be structures that are of the form of the Kuroda identities. The filter is designed by the following steps:

- Lumped element low pass prototype (from tables, typically)
- Convert series inductors to series stubs, shunt capacitors to shunt stubs
- Add l/8 lines of $Z_O = 1$ at input and output
- Apply Kuroda identity for series inductors to obtain equivalent with shunt open stubs with $\lambda/8$ lines between them
- Transform design to 50Ω and f_C to obtain physical dimensions (all elements are $\lambda/8$).

The completed filter in microstrip form looks like this:



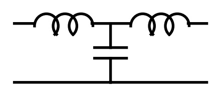
Although the lumped element filter has only lowpass response, the periodic nature of distributed elements results in harmonic responses because of the periodic nature of the structure. In this case, there are responses centered on $4nf_c$, where n can take any positive integral value. This is a general characteristic of distributed-element filters, and can be a problem if unanticipated in system designs.

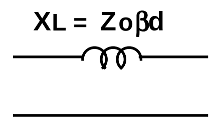
This general characteristic of periodic structures can be a serious problem in the application of distributed-element filters in microwave systems. It is often an overlooked artifact of filters constructed entirely from transmission line elements.

Stepped Impedance Low Pass Filters

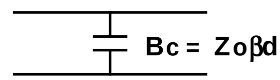
Consider the T-section equivalent circuit of a short section of transmission line, as determined from conversion of the ABCD parameters to Z parameters to identify the individual elements.

T-section equivalent circuit of transmission line section βd and Z_o.





For high Z_o and small βd the equivalent circuit becomes $X_L = Z_o\beta d$



For low Z_o and small βd, the equivalent circuit becomes $B_L = Z_o\beta d$

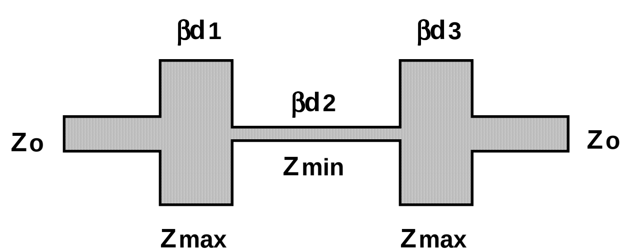
So we can use short sections of transmission line to realize a lumped element low pass filter in transmission line format. In order to use this approximation, we need to know the highest and lowest feasible transmission line impedances, Z_{max} and Z_{min}.

Then the length of each transmission line element will be

$$\beta d = \frac{LR_o}{Z_{max}} \quad \text{for inductive elements and}$$

$$\beta d = \frac{CZ_{max}}{R_o} \quad \text{for capacitive elements,}$$

where R_o is the filter impedance and L and C are the normalized element values of the lowpass prototype.



As a worked example, consider the design of a maximally flat three-section lowpass filter that has a cutoff frequency of f_c = 1 GHz. The highest practical impedance is Z_{max} = 120 ohms and the lowest practical impedance is Z_{min} = 12 ohms. Assume we have determined from Figure 8.26 of Pozar[2] that this will provide the required outband attenuation.

Using prototype values from Pozar's Table 8.23, the L and C values and corresponding βd are

Element	g_n	C_n	L_n	$\beta d = \frac{CZ_{max}}{R_o}$	$\beta d = \frac{LR_o}{Z_{max}}$
1	1.0	1.0		14°	
2	2.0		2.0		48°
3	1.0	1.0		14°	

The physical lengths of the transmission line sections are calculated by using the wavelength in the transmission line at f_c. This type of filter can be realized in microstrip format, and is also made in tubular form, with the high and low impedance consisting of different diameter conductors. In the tubular filter, some provision must be made to support the filter center conductor in the tubular outer conductor; typically the low impedance (larger diameter) sections are wrapped in Teflon ($\varepsilon_r = 2.08$) sheet, which supports the filter and also lowers Z_{min} without greatly affecting Z_{max}.

If we assume a microstrip structure with $e_{eff} = 9$,

$$v = \frac{c}{\sqrt{9}} = 10^8 \text{ m/s and } \lambda g = \frac{v}{f} = \frac{10^8}{10^9} = 0.1 \text{ m, so the lengths are}$$

$$d_1 = d_3 = 0.1\frac{14}{360} = 3.8\text{x}10^{-3} \text{ m} = 3.8 \text{ mm and } d_2 = 0.1\frac{48}{360} = 1.3\text{x}10^{-2} \text{ m} = 13 \text{ mm.}$$

<u>Bandpass and Bandstop Filters</u>

A useful form of bandpass and bandstop filter consists of $\lambda/4$ stubs connected by $\lambda/4$ transmission lines. Consider the bandpass filter here

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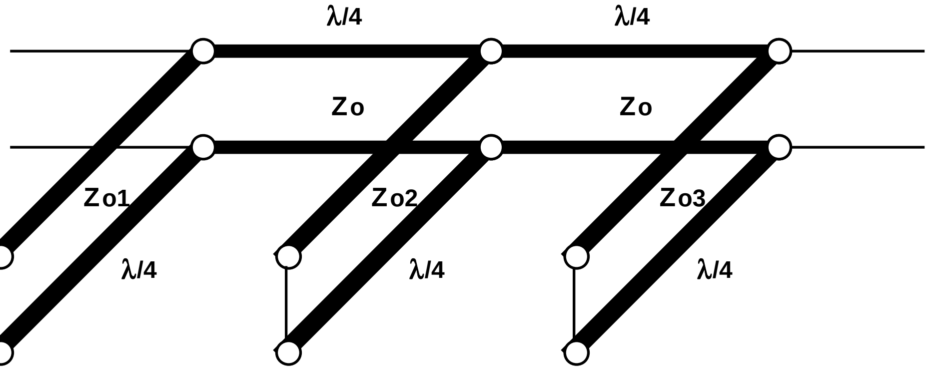
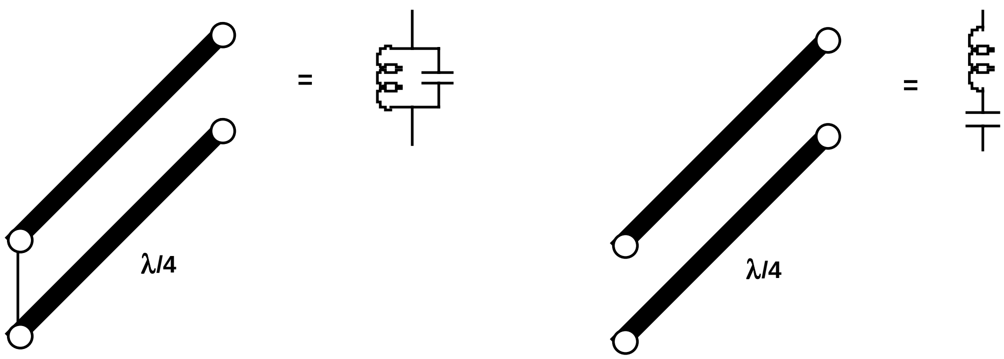
[2] Pozar, D., *Microwave Engineering*, 2nd Edition, J. Wiley, 1998, pg. 449, 450

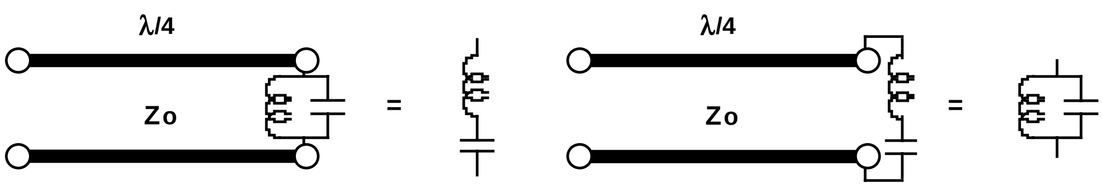


This filter can also be configured as a bandstop filter by using open rather than shorted stubs. While it's easy to see that this filter will pass the center frequency for which the lines are all $\lambda/4$, we would like to be able to design such a filter using lumped element prototypes.

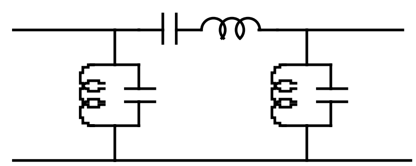
Recall that the equivalent circuit of a quarter wave transmission line resonator is, for shorted or open circuit termination, a parallel or series tuned resonant circuit, as shown:



But it is important to note that a parallel tuned circuit is transformed through a $\lambda/4$ line to the impedance of a series tuned circuit, and vice versa. This allows us to determine the equivalent circuit of the transmission line filter.



The quarter wave sections transform the center shunt parallel resonant circuit admittance to a series impedance that is a series resonant circuit.

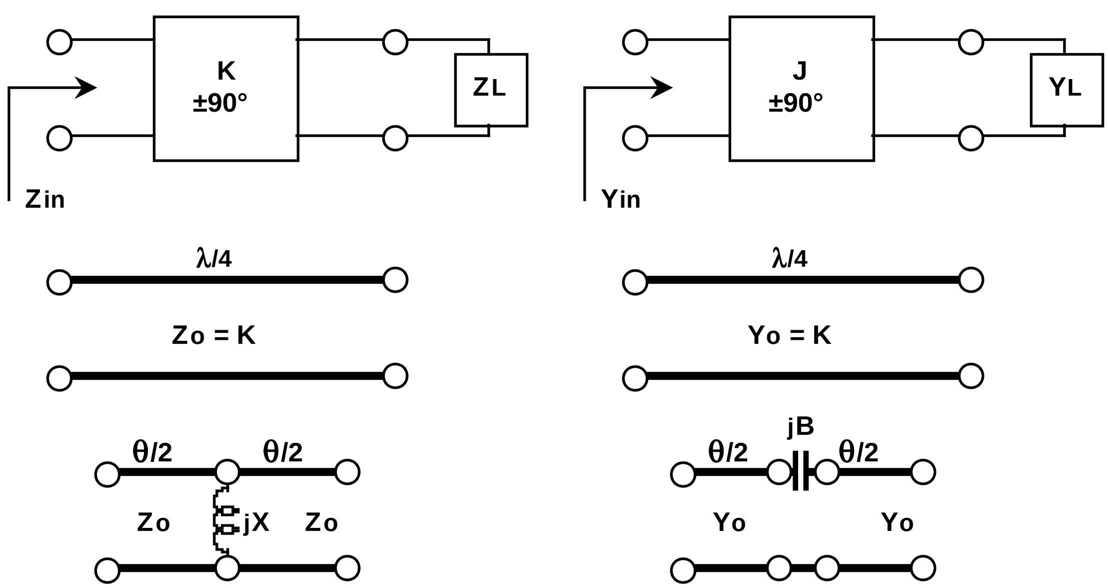


Thus, the equivalent circuit of the bandpass filter using quarter wave lines is the same as the prototype lumped element filter that is created through the customary transformation from lowpass to bandpass prototype filters. Using the known relationships between transmission line Z_o and the L and C of the equivalent resonance, we can identify the relationships between the required L and C of the prototype circuit and the Z_o we need for the shunt stubs.

Impedance and Admittance Inverters

In this process we've uncovered another "magic bullet" comparable to the Kuroda identities, only involving $\lambda/4$ rather than $\lambda/8$ lines. Quarter wave lines can transform series connected element to shunt, and vice versa. There are also combinations of transmission line and lumped elements that perform the same function. Transmission line networks having this general property are called impedance and admittance inverters.

The quarter wave and alternative implementation are shown here for both functions:



For the impedance inverter, $Z_{in} = K^2/Z_L$

For the 90° line, $K = Z_o$

For the lumped element implementation,

$K = Z_o \tan|\theta/2|$

$$X = \frac{K}{1-(K/Z_o)^2}$$

$$\theta = -\tan^{-1}\frac{2X}{Z_o}$$

For the admittance inverter, $Y_{in} = J^2/Y_L$

For the 90° line, $J = Y_o$

For the lumped element implementation,

$J = Y_o \tan|\theta/2|$

$$B = \frac{J}{1-(J/Y_o)^2}$$

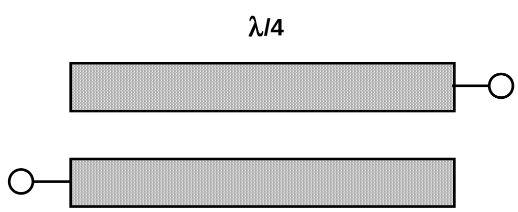
$$\theta = -\tan^{-1}\frac{2B}{Y_o}$$

The transmission line lengths θ/2 are generally negative in this circuit, but this is not a problem if the lines can be absorbed by reducing the length of connecting transmission lines on either side as is often the case in filter embodiments.

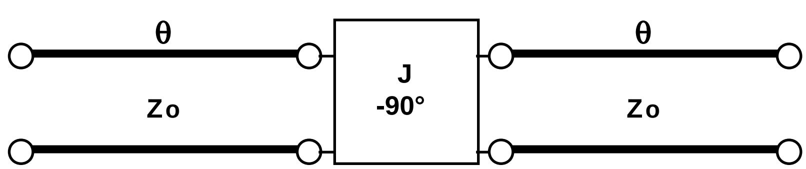
Coupled Line Filters

With the added tool of the impedance or admittance inverter, we can analyze and design a number of transmission line filters. As we have seen in connection with directional couplers, coupled transmission lines have frequency sensitive coupling, and can be analyzed by the even-odd mode method.

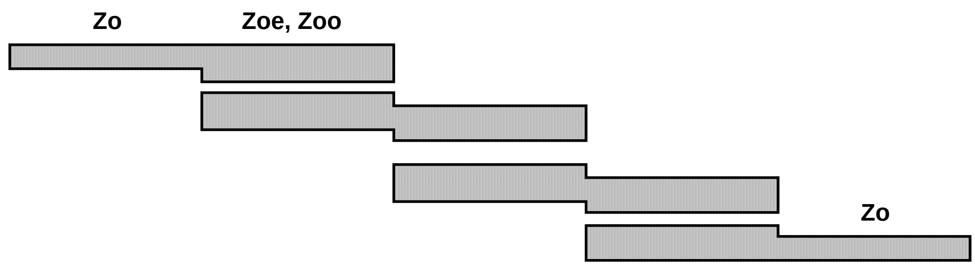
The result of this analysis is tabulated in Table 8.8 of Pozar, and we can see that there are among the less useful permutations several that have bandpass characteristics. In particular, the configuration that represents coupled λ/2 open lines is the easiest to construct in microstrip and stripline.



The equivalent circuit of two coupled λ/4 open lines can be shown to be as depicted here:



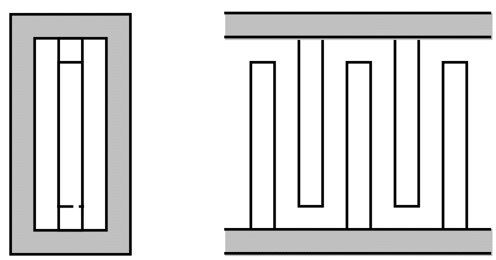
So we can see that a structure of a number of coupled lines will admit to an equivalent circuit of alternating series and parallel resonant circuits, and the design parameters of the prototype filter can be imposed onto the structure of parallel coupled lines.



In microstrip or stripline, the transmission line conductors of the coupled line filter take the form shown here, with the offsets between connected λ/4 sections added to permit seeing the individual coupled line pairs.

Interdigital Filters

If the coupled line configuration with short-circuit termination of the λ/4 lines is chosen, a useful filter form can be constructed in a rectangular transmission structure. Because the lines are shorted at opposite ends, the structure takes the form of interlaced fingers, and is called an interdigital filter.

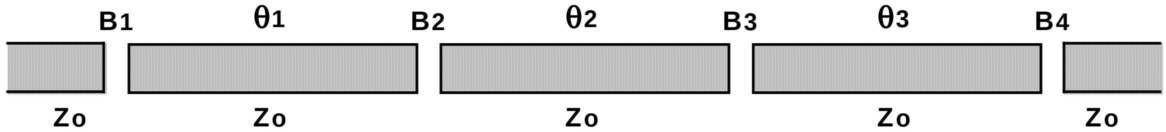


The interdigital filter structure typically takes the form shown here, with the casing and fingers typically silver plated machined aluminum; another form uses cylindrical pins pressed into a housing, but there can be problems with the joint at the high current end of the transmission line fingers:

Filter with Capacitively Coupled Resonators

Essentially the same filter structure as the coupled line filter can be made using the alternative form of impedance inverter. The length of the transmission lines must be shortened to account for the negative-length line required for the admittance inverter between each near-half-wave section.

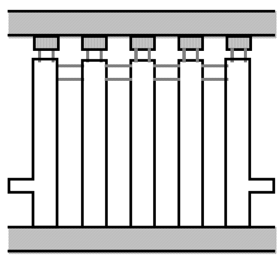
This configuration, the capacitively coupled resonator filter, takes the form shown here:



It can be seen that the equivalent circuit can incorporate the short admittance inverter, and that the resonators can be shortened to incorporate the negative lengths required. This filter can be designed by reference to a lumped element prototype, using the equivalent circuits of the near-half-wave resonators and identifying corresponding parameters.

Comb-Line Filters

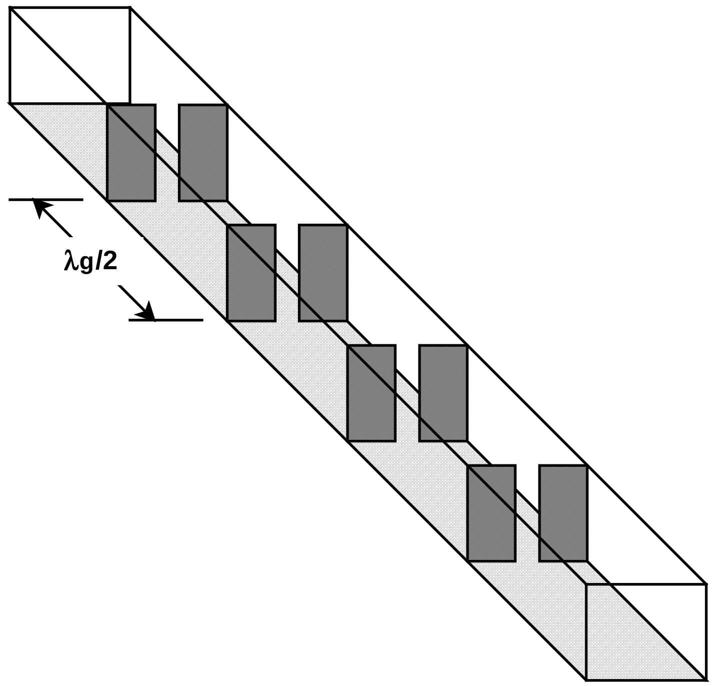
A very useful filter embodiment incorporates capacitively coupled quarter wave resonators, often with additional lumped capacitive tuning screws for fine adjustment. This form is called the comb-line filter, since all the strip transmission lines are grounded at the same end. Although the analysis of Pozar Table 8.8 would imply that this is an all-stop network, the effect of capacitive loading creates a very functional filter.



One major benefit of the combline filter is that, uniquely among the forms shown above, it can be designed so that the interplay of distributed and lumped elements can be used to eliminate the higher order passbands. As we will see in later lectures, this is a very significant advantage for large-signal circuits that have substantial harmonic energy.

Waveguide Discontinuities and Filters

The low loss and high power handling characteristics of waveguide lend themselves to the use of waveguide in specialized filters. As we have seen, waveguide discontinuities can be constructed to provide equivalent susceptance, and the transmission line equivalent circuits can be used to design specific filter passbands that can be derived from lumped element equivalents.



Waveguide filters generally employ iris-coupled resonant sections in a pattern similar to the capacitive coupled resonator filter. The waveguide can be cut with a saw and the iris material soldered in before plating. As with other metallic filters, it is not unusual for a designer to take advantage of the opportunity to have tuning screws to accommodate small tolerance variations in the filter.

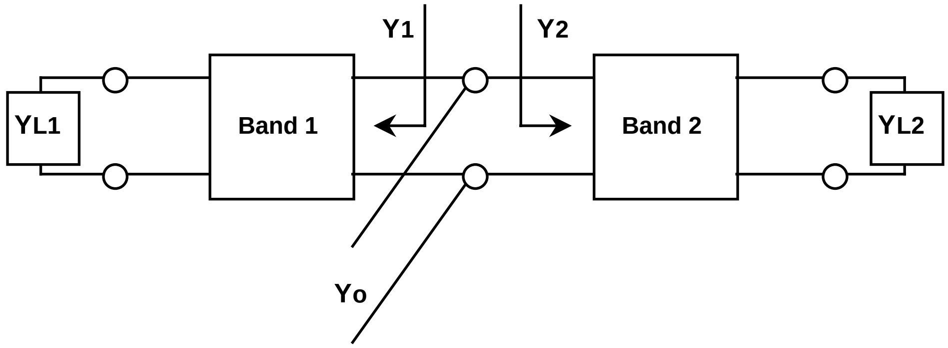
Impedance Matching Filters and Diplexers

Because there is no restriction on the output admittance of the prototype filter, there are occasions when it is appropriate to use a filter designed to provide bandpass matching to a lower or higher impedance. This technique permits tailoring of the passband so that the best possible match is obtained, subject to the limitations of the Bode-Fano limit.

The requirements are that Y_1 and Y_2 be as follows:

In Band 1 $Y_1 = Y_o$ $Y_2 = 0$
In Band 2 $Y_1 = 0$ $Y_2 = Y_o$

The line lengths to the junction point can be used to cancel reactances in the input impedances of the diplexer filters, and this technique can be extended to multiple bands.

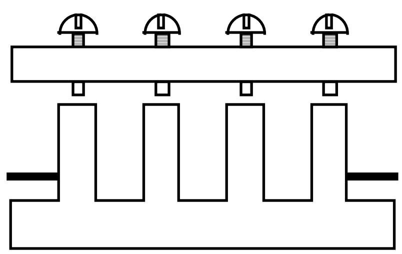


It is also common to combine multiple filters in such a way as to direct certain bands of frequencies losslessly to different output ports. Because filters are reciprocal, this may also be used to combine different input frequency bands without loss. In general, the technique is to use complementary lowpass and highpass filters which can be connected at a common point such that the impedance at that point is matched over the full range of frequencies. This technique can be extended to the combining of multiple bandpass filters.

A common example is the combiner or separator for VHF and UHF television signals so that separate antennas may be used for each band.

The Dishall Tuning Method

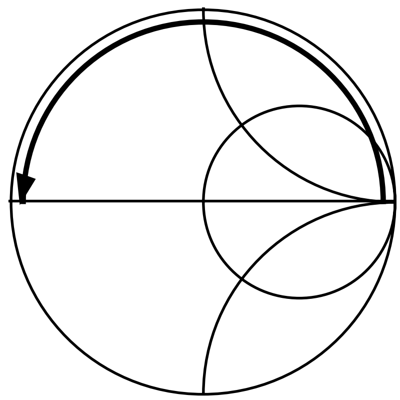
A brief subject of practical interest is the tuning resonators and filters by the Dishall method[3]. This is a method of tuning multisection filters by observing only the phase of the reflection coefficient at the center frequency of one of the two ports, with the other terminated.

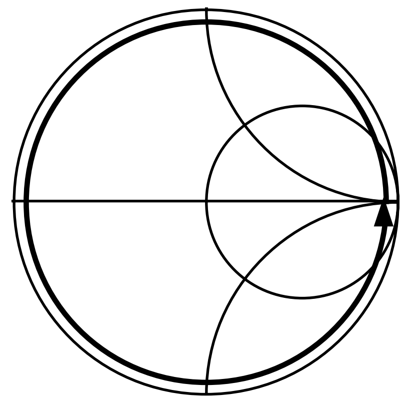


Consider a filter such as a comb-line filter which has been designed but must now be tuned. If all the tuning screws are shorted to their respective resonators, the output admittance at, say, the input port plane is very high. As that resonator is tuned, the reflection coefficient stays high because the rest of the filter is still shorted, but the phase of the reflection coefficient can range through a 180° range. If we stop at 90° phase

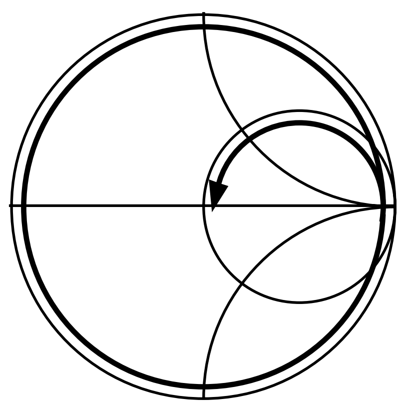
[3] Dishall, S., *Proc IRE*, Nov. 1951, Zverev, A. , *Handbook of Filter Synthesis* and Matthaei, Young and Jones, Ch. 8

angle, we have the correct tuning of the first resonator. Similarly, we can tune the second resonator to another 90° of phase shift, and so on until we come to the last resonator.





When we tune the last resonator, we see the load at the output of the filter. But we can still get the proper tuning by observing the additional 90° phase change.



A filter tuned by the Dishall method will be very close to the design reflection and passband characteristics. This is one of the tricks of the trade that has gone unnoticed for some time.