Mirowave Transistors

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>GaAs FET</th>
<th>GaAs MESFET</th>
<th>Silicon Bipolar</th>
<th>GaAs HBT</th>
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Modeling method
- $S$-parameters: up to meas. freqs.
- Equivalent ckt
  - freq. extrapolation
  - physical

GaAs FET

Physical structure

Intrinsic EC model (no pad)

$H_{21} = \frac{I_2}{I_1} = \frac{g_m V}{\omega C_{gs} V} = \frac{g_m}{\omega C_{gs}}$

$f_T = \frac{g_m}{2\pi C_{gs}}$

Biasing scheme
Power Gains

**Power Gain Definition**

- **Power gain** (G)
  
  \[ G = \frac{P_{av}}{P_{in}} \]

- **Available gain** (Ga)
  
  \[ G_a = \frac{P_{av}}{P_{in}} \]

- **Transducer Power gain** (Gr)
  
  \[ G_r = \frac{P_{L}}{P_{in}} \]

\[ \Gamma_{in} \quad \Gamma_{L} \quad \Gamma_{out} \]

**Available Gain**

\[ G_a = \frac{|S_{21}|^2 (1-|\Gamma_L|^2)}{1-S_{22} \Gamma_L} \]

**Transducer Power Gain**

\[ G_r = \frac{|S_{21L}|^2 (1-|\Gamma_L|^2) }{1-S_{22} \Gamma_L} \]

**Available Gain & Transducer Power Gain**

\[ G_{tot} = G_a \cdot G_r \]

Amplifier Gain - S-parameter Approach

**Formula**

\[ G = \frac{|S_{21}|^2 (1-|\Gamma_L|^2)}{1-S_{22} \Gamma_L} \]

\[ G_a = \frac{|S_{21}|^2 (1-|\Gamma_L|^2)}{1-S_{22} \Gamma_L} \]

\[ G_r = \frac{|S_{21L}|^2 (1-|\Gamma_L|^2) }{1-S_{22} \Gamma_L} \]

\[ G_{tot} = G_a \cdot G_r \]

**Available Gain & Transducer Power Gain**

\[ G_a = \frac{|S_{21}|^2 (1-|\Gamma_L|^2)}{1-S_{22} \Gamma_L} \]

\[ G_r = \frac{|S_{21L}|^2 (1-|\Gamma_L|^2) }{1-S_{22} \Gamma_L} \]

\[ G_{tot} = G_a \cdot G_r \]
Amplifier Gain - from Equivalent Circuit

Matching condition (Assume $C_{gd}=0 \rightarrow S_{12}=0$)

$$Z_a = Z_b \Rightarrow X = \frac{-1}{\omega C_{ph}}$$

$$Z_{out} = Z' \Rightarrow B = -\omega C_{ph}$$

$$G_{iv} = \frac{1}{8} \left[ \frac{V_i}{V_o} \right] R_{in} = \frac{R_{in}}{4\omega R C_{ph}^2} = \frac{R_{in}}{4\omega} \left( \frac{f_0}{f} \right)^2$$

$$\Rightarrow 6dB/\text{oct roll-off}$$

Maximum frequency of oscillation

$$f_{max} = \frac{f_0}{2} \left( \frac{R_{in}}{R_{o}} \right) \rightarrow \text{most important parameter for Amp., Osc.}$$

Stability

Stability Circle

- $|\Gamma_a|=1$ circle in $\Gamma_S$ - plane
  - $\Rightarrow$ output stability circle
- $|\Gamma_a|=1$ circle in $\Gamma_L$ - plane
  - $\Rightarrow$ input stability circle

for $|\Gamma_a|=1$

$$\Gamma_a - \frac{(S_{22} - \Delta S_{12})^*}{S_{12}} = \frac{|S_{12}|^2}{|S_{22}|^2} - |\Gamma_a|^2$$

$$\Rightarrow \text{Radius}(R_L) = \frac{|S_{22}|}{|S_{12}|}$$

$$\Rightarrow \text{Center}(C_L) = \frac{(S_{22} - \Delta S_{12})^*}{|S_{22}|^2}$$
### Stability Circle

**Stable region? (output)**

1. \(|S_{11}| < 1\)
2. For \(\Gamma_L = 0\), \(\Gamma_i = S_{11} < 1\)
3. Stable for region containing \(\Gamma_L = 0\)
4. \(|S_{11}| > 1\)
5. Stable for region excluding \(\Gamma_L = 0\)

---

### Amplifier Stability

**Stability**

1. **Unconditional Stability**
   
   \[|\Gamma_{in}|, |\Gamma_{out}| < 1\]

2. **Conditional Stability**
   
   \[|\Gamma_{in}|, |\Gamma_{out}| < 1\]

**Conditions (iff)**

\[K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{11}S_{22}|} > 1\]

where \(\Delta = S_{12}S_{22} - S_{11}S_{21}\)

**Conditional Stable**

\[\Rightarrow \text{Need to choose } \Gamma_{i}, \Gamma_{l} \text{ for stable region}\]
Narrowband Amplifier Design

Maximum gain
\[
\begin{align*}
\Gamma_{in} &= \Gamma_s^* \\
\Gamma_{out} &= \Gamma_L^* \\
G_{max} &= \frac{1}{1-|S_{21}|^2} \left(1-|\Gamma_d|^2\right) \\
&= G_s \cdot G_0 \cdot G_L \\
\end{align*}
\]

Condition for simultaneous match
\[
\begin{align*}
\Gamma_s^* &= S_{11} + \frac{S_{21} S_{22} \Gamma_s}{1-S_{22} \Gamma_s} \\
\Gamma_L^* &= S_{22} + \frac{S_{21} S_{22} \Gamma_L}{1-S_{22} \Gamma_L} \\
\end{align*}
\]

where
\[
\begin{align*}
B_1 &= 1 + |S_{21}|^2 - |S_{22}|^2 - |\Gamma_d|^2 \\
C_1 &= S_{11} - \Delta S_{\Gamma_s} \\
B_2 &= 1 + |S_{21}|^2 - |S_{22}|^2 - |\Gamma_d|^2 \\
C_2 &= S_{22} - \Delta S_{\Gamma_L} \\
\end{align*}
\]

Unilateral Case

Unilateral Transducer Gain
\[
\begin{align*}
\Gamma_{in} &= S_{11}, \quad \Gamma_{out} = S_{22} \\
G_{TU} &= \frac{|S_{21}|^2 (1-|\Gamma_s|^2) (1-|\Gamma_L|^2)}{|1-S_{11} \Gamma_s|^2 |1-S_{22} \Gamma_L|^2} \\
\end{align*}
\]

Condition for Simultaneous Match
\[
\begin{align*}
\Gamma_s^* &= S_{11}, \quad \Gamma_L^* = S_{22} \\
\end{align*}
\]

Maximum Unilateral Gain
\[
\begin{align*}
G_{TU_{max}} &= \frac{1}{1-|S_{11}|^2} \left|\frac{|S_{21}|^2}{1-|S_{22}|^2}\right| \\
\end{align*}
\]
Mismatched Amp Design - Gain Circles (unilateral)

Gain for unilateral case
\[ G = G_s |S_{21}|^2 G_L \]
\[ G_s = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} \]
\[ G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \]

Maximum unilateral gain
\[ G_{S\text{max}} = G_s |S_{21}|^2 |G_{L\text{max}}| \]
\[ G_{L\text{max}} = G_L |S_{11}|^2 |G_{S\text{max}}| \]

Gain factor \( g_s, g_L \)
\[ G = G_{S\text{max}} |S_{21}|^2 G_{L\text{max}} \]
\[ G_s = \frac{G_s}{G_{S\text{max}}} \]
\[ G_L = \frac{G_L}{G_{L\text{max}}} \]

Gain circles
For given \( g_s \), find \( \Gamma_S \)
\[ \Gamma_S = \frac{G_s}{G_{S\text{max}}} = \frac{1 - |\Gamma_s|^2}{|1 - |S_1|^2|} \]
\[ \Rightarrow \Gamma_S = \frac{1 - \frac{g_s}{G_{S\text{max}}}}{1 - \frac{1}{|S_1|^2}} \]

\( \Rightarrow \) Circles!
- \( \Gamma_{\text{center}} : \) on the line to \( S^{*}_{11} \)
- \( g_s = 0 \Rightarrow \Gamma_S = 1 \)
- \( 0 \text{dB} \) circle cross the center

Gain Circle and Bandwidth

1. Exact match design
2. Mismatch design

Choose \( \Gamma_S \) and \( \Gamma_L \) close to the center of Smith chart

for max bandwidth
Low Noise Amplifier Design

Noise figure of 2-port device

\[ F = F_{\text{min}} + \frac{R_L}{G_s} |Y_s - Y_{\text{opt}}|^2 \]

where \( Y_s = G_s + jB_s \cdot Y_{\text{opt}} = G_{\text{opt}} + jB_{\text{opt}} \)

4 noise parameters

- \( R_{\text{min}} \)
- \( R_h \)
- \( G_{\text{opt}} \)
- \( B_{\text{opt}} \)

S-parameter representation

\[ F = F_{\text{min}} + \frac{4R_L}{Z_0} \frac{|\Gamma_s - \Gamma_{\text{opt}}|^2}{(1 - |\Gamma_s|^2) |\Gamma_{\text{opt}}|^2} \]

Noise figure parameters

\[ N = \frac{F - F_{\text{min}}}{\Delta F} |1 + \Gamma_{\text{opt}}|^2 \]

determined by noise parameters

Constant Noise figure circles

\[ N = \frac{\Gamma_s - \Gamma_{\text{opt}}}{1 - |\Gamma_s|^2} \]

\[ \Gamma_s = \frac{\Gamma_{\text{opt}}}{N + 1} \]

\[ \frac{N(N + 1 - |\Gamma_{\text{opt}}|^2)}{(N + 1)^2} \]

* Circle center on a straight line from the center to \( \Gamma_{\text{opt}} \)

Low Noise Amplifier Design (cont'd)

Amplifier design
- RL, Stability, Bandwidth

LNA design
- RL, Stability, NF, Bandwidth

Compromise between NF, RL, Stability and Gain
Oscillator - Types

Two types of VCO
- Transmission type: amplifier mode ⇒ straightforward
- Reflection type: negative resistance mode: less feed structure ⇒ potentially higher Q factor

One - port Negative Resistance Oscillator

Negative Resistance Device

\[ I \quad V \]

\[ \text{Gunn diode, IMPATT, TUNNET} \]

Circuit

\[ KVL \]
\[ (Z_L + Z_n)I = 0 \]
\[ \Rightarrow R_L + R_n = 0 \]
\[ X_L + X_n = 0 \]

S-parameter representation

\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-Z_m - Z_0}{Z_m + Z_0} = \frac{1}{\Gamma_L} \]
\[ \Rightarrow \Gamma_L \cdot \Gamma_{in} = 1 \text{ at s.s} \]

Stable oscillation with perturbations in I or frequency, go back to original oscillation status

\[ Z_m(I, \omega), Z_L(\omega) \]
\[ \Rightarrow \frac{\partial (X_L + X_m)}{\partial \omega} \gg 0 \]
\[ \therefore \text{High Q resonator needed} \]
Transistor Oscillator

Design procedure

1. Choose $\Gamma_T$ for max $\Gamma_{\text{out}} (>1)$
   
   $Z_{\text{out}} = R_{\text{out}} + jX_{\text{out}}$
   
   $\Rightarrow$ Maximize $|Z_{\text{out}}| (\text{negative})$

2. Choose $Z_L = R_L + jX_L$
   
   $R_L = \frac{R_{\text{out}}}{3}$ (for max power)
   
   $X_L = -X_{\text{out}}$

Simultaneous Oscillation - output oscillator

$\Gamma_T = \Gamma_{\text{out}}$

$\Gamma_{\text{out}} = S_{22} + \frac{S_3\Gamma_L}{1-S_1\Gamma_L} = S_{22} - \Delta \Gamma_L$

$\Gamma_T = \frac{1-S_3\Gamma_L}{S_1 - \Delta \Gamma_L}$

$\Gamma_T = \frac{S_3\Gamma_L}{1-S_2\Gamma_L}$

$\Rightarrow \Gamma_T \Gamma_{\text{in}} = 1$

$\Rightarrow$ Input oscillation

(DLarge signal S-parameter)

DRO: Dielectric Resonator Oscillator

$\Gamma$ at the resonator plane

$\Gamma = \frac{(Z_0 + N^2 R) - Z_0}{Z_0 + N^2 R + Z_0} = \frac{g}{1+g}$

$\therefore$ Measure $\Gamma \rightarrow$ find $g \rightarrow$ find $N^2 R$
DRO Design

Design Procedure

1. Find $\Gamma_L$ for max $|R_{out}|$
2. Design output matching circuit
   \[ Z_T = \frac{R_{out}}{3} - jX_{out} \]

- Transform $\Gamma_L$ to $\Gamma_L'$
  \[ \Gamma_L' = \Gamma_L e^{j\phi} \]
- Find $Z_L'$
  \[ Z_L' = Z_0 \frac{1 + \Gamma_L'}{1 - \Gamma_L'} \]
- Find $g$
  \[ g = \frac{Z_L'}{Z_0} \quad (\text{note: } Z_L = Z_0) \]

$\Gamma_{out}$ vs frequency (effect of DR)