

**Assignment #7**

- 4.5 *Employ the conversion table for the different parameter representations of the two-port network and find the h-matrix representation for a Darlington Pair shown in Figure 4-7 under the assumption that the transistors are specified by the same h-matrices derived in Example 4-2.*

As a first step replace Q1 connected to R by a cascade of two networks. To find a matrix representation of this connection we have to convert the h-parameter matrix for the transistor into an ABCD form and multiply it by the ABCD representation of the resistor

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{Q1+R} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{Q1} \begin{bmatrix} A & B \\ C & D \end{bmatrix}_R = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{Q1} \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}$$

The result of this operation has to be converted back to h-matrix form and added to the h-parameter matrix of Q2. The result is:

$$[h] = [h]_{Q2} + \begin{bmatrix} \frac{R\Delta h + h_{11}}{Rh_{22} + 1} & \frac{h_{12}}{Rh_{22} + 1} \\ \frac{h_{21}}{Rh_{22} + 1} & \frac{h_{22}}{Rh_{22} + 1} \end{bmatrix}_{Q1}$$

where  $\Delta h$  is the determinant of the h-parameter matrix for Q1.

- 4.9 *Find the ABCD parameters for a generic three element pi-network, as depicted in Figure 4-2.*

One way to solve this problem is to directly evaluate each parameter by using the definition for the ABCD matrix. Alternately, we can use our knowledge of the ABCD representations for the individual circuit components as given in Table 4-1. With reference to Figure 4-2 we can write

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Z_A^{-1} & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_B \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Z_C^{-1} & 1 \end{bmatrix} = \begin{bmatrix} 1 + \frac{Z_B}{Z_C} & Z_B \\ \frac{Z_A + Z_B + Z_C}{Z_A Z_C} & 1 + \frac{Z_B}{Z_A} \end{bmatrix}$$

- 4.10 Compute the ABCD parameters for an RF transformer with turn ratio  $N=N_1/N_2$ , where  $N_1$  and  $N_2$  are the number of turns in the primary and secondary, respectively.

Since the voltages in the transformer are related as  $v_1/N_1 = v_2/N_2$ , we can use the definition of the ABCD matrix and write

$$A = \left. \frac{v_1}{v_2} \right|_{i_2=0} = \frac{N_1}{N_2} = N$$

Because of  $v_1/N_1 = v_2/N_2$  for the ideal transformer the condition  $v_2 = 0$  immediately leads to  $v_1 = 0$ . Therefore

$$B = \left. \frac{v_1}{-i_2} \right|_{v_2=0} = 0.$$

Furthermore since there are no losses in an ideal transformer, the input power is equal to the output power

$$P_1 = P_2 \Rightarrow v_1 i_1 = -v_2 i_2$$

Here we took in to account that the direction of the current flow in the secondary winding of the transformer is opposite to the current direction at port 2. Taking into account the relation between the voltages at ports 1 and 2, we can write

$$i_1 = -\frac{v_2}{v_1} i_2 = -\frac{N_2}{N_1} i_2 = -\frac{i_2}{N}$$

Using the relations between the input and output currents we can calculate the remaining two parameters as

$$C = \left. \frac{i_1}{v_2} \right|_{i_2=0} = 0, \quad D = \left. \frac{i_1}{-i_2} \right|_{v_2=0} = \frac{1}{N}.$$

- 4.14 Find the forward gain of the circuit in Example 4-8.

From Equation 4.47 we observe that the power at port 1 is computed as

$$P_1 = \frac{1}{2} |a_1|^2 (1 - |\Gamma_{in}|^2)$$

Following an identical derivation we can obtain the power delivered to the load as

$$P_L = \frac{1}{2} |a_L|^2 (1 - |\Gamma_L|^2) = \frac{1}{2} |b_2|^2 (1 - |\Gamma_L|^2)$$

Our objective is to find the power gain which is equal to the ratio of the power delivered to the load over the input power:

$$G = \frac{P_L}{P_1} = \frac{|b_2|^2 (1 - |\Gamma_L|^2)}{|a_1|^2 (1 - |\Gamma_L|^2)}$$

With reference to Figure 4-23(b) we can write  $b_2 = S_{21}a_1 + S_{22} \Gamma_L b_2$  or  $b_2 = \frac{S_{21}}{1 - S_{22}\Gamma_L} a_1$

Substitution yields the gain of the circuit

$$G = \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)}{|1 - S_{22}\Gamma_L|^2 (1 - |\Gamma_{in}|^2)}$$

$$\text{where } \Gamma_{in} = S_{11} + \frac{S_{12}S_{21}}{1 - S_{22}\Gamma_L} \Gamma_L.$$

- 4.15 Given that the input of an amplifier has a VSWR = 2 and the output is given by VSWR = 3, find the magnitudes of the input and output reflection coefficients. What does your result mean in terms of  $S_{11}$  and  $S_{22}$  ?

$$\text{Since the input VSWR} = 2, |\Gamma_{in}| = \frac{\text{VSWR}_{in} - 1}{\text{VSWR}_{in} + 1} = \frac{1}{3}$$

$$\text{Similarly the output VSWR is } |\Gamma_{out}| = \frac{\text{VSWR}_{out} - 1}{\text{VSWR}_{out} + 1} = \frac{1}{2}$$

The output reflection coefficient can be represented as

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}}{1 - S_{22}\Gamma_L} \Gamma_L$$

Since the system is symmetric we can write an almost identical expression for the output reflection coefficient of the amplifier

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}}{1 - S_{11}\Gamma_S} \Gamma_S$$

In the general case, where all S-Parameters and source and load reflection coefficients are nonzero, we cannot estimate the values of the  $S_{11}$  and  $S_{22}$  parameters of the amplifier solely based on the values of the input and output reflection coefficients. However, if the load and source reflection coefficients are equal to zero, i.e. load and source impedances are matched to the characteristic impedance of the system, then

$$|S_{11}| = |\Gamma_{in}| \quad \text{and} \quad |S_{22}| = |\Gamma_{out}|$$