ELEC 412 RF

& Microwave Engineering

Fall 2004

Lecture 2

Transmission Line Analysis

- Propagating electric field

Space factor

$$E_X = E_{0X}\cos(\omega t - kz)$$

Time factor

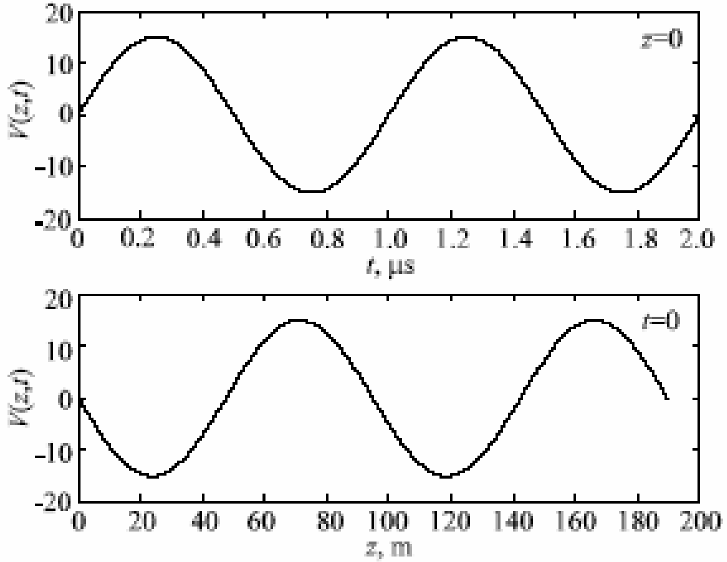
- Phase velocity

$$v_p = \lambda\ f = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{\sqrt{\varepsilon_r}}$$

- Traveling voltage wave

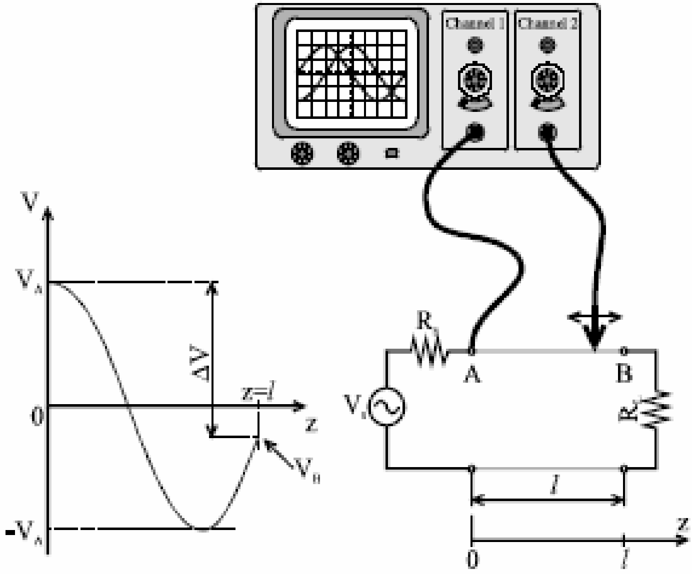
$$V(z,t) = E_{0X}\frac{\sin(\omega t - kz)}{k}$$

High frequency implies spatial voltage distribution



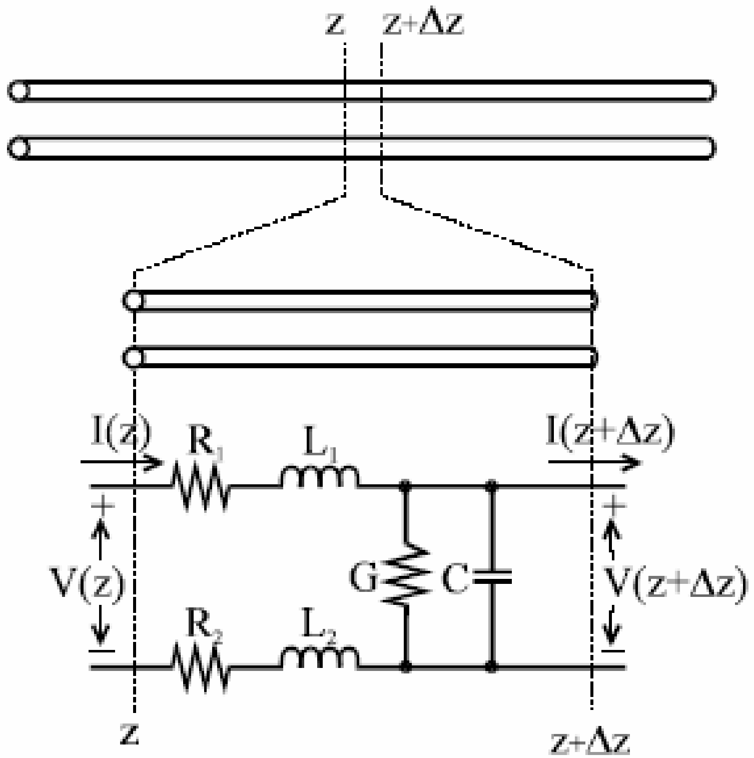
- Voltage has a time and space behavior
- Space is neglected for low frequency applications
- For RF there can be a large spatial variation

Generic way to measure spatial voltage variations



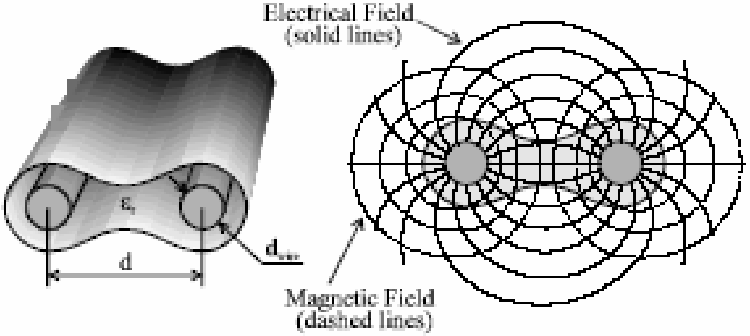
- For low frequency (1MHz) Kirchhoff's laws apply
- For high frequency (1GHz) Kirchhoff's laws do **not apply** anymore

Kirchhoff's laws on a microscopic level



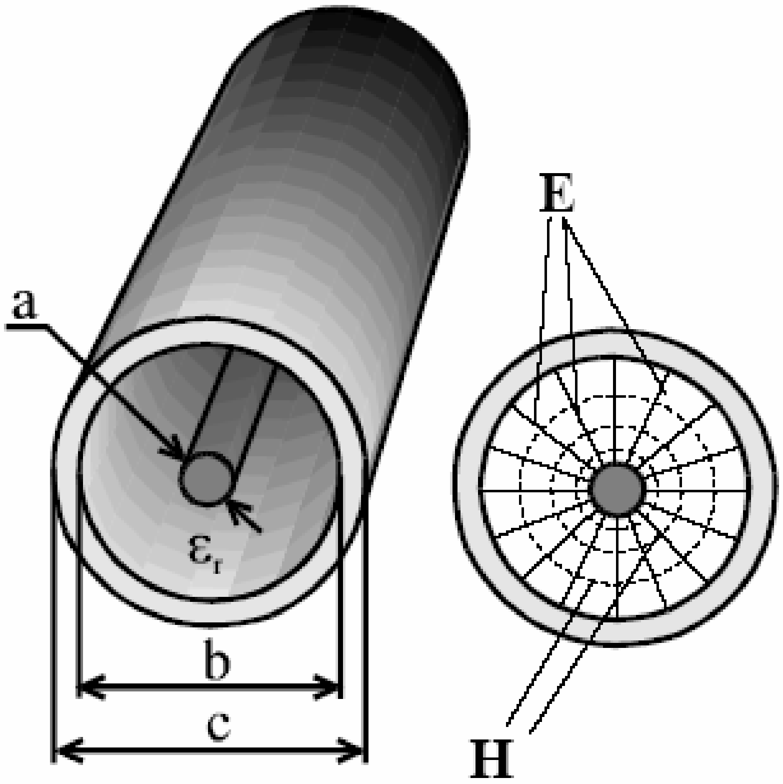
- Over a differential section we can again use basic circuit theory
- Model takes into account line losses and dielectric losses
- Ideal line involves only L and C

Example of transmission line: **Two-wire line**



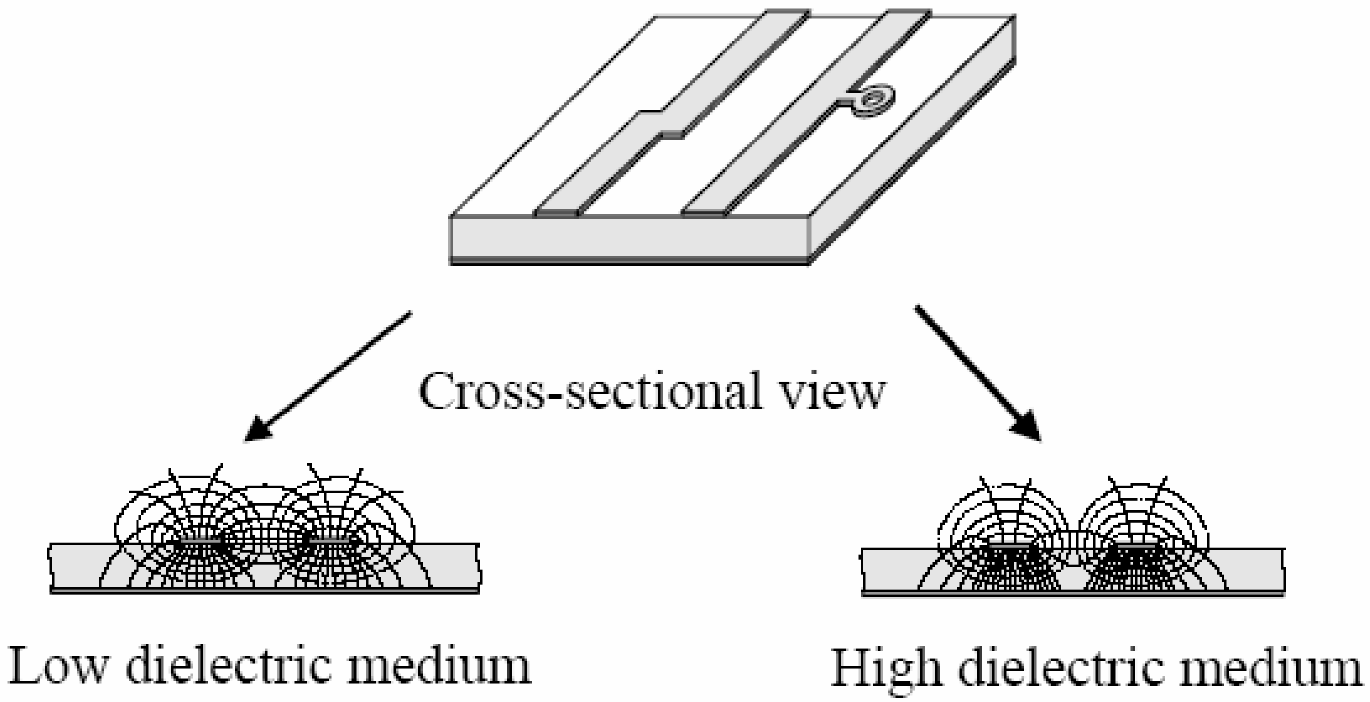
- Alternating electric field between conductors
- alternating magnetic field surrounding conductors
- dielectric medium tends to confine field inside material

Example of transmission line: **Coaxial cable**



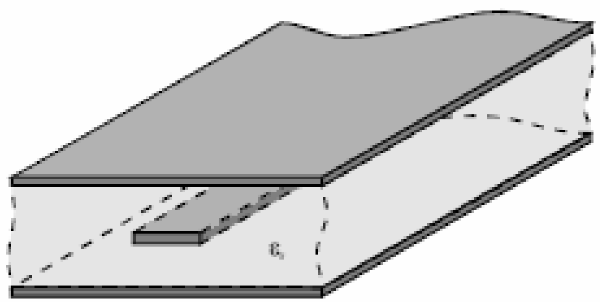
- Electric field is completely contained within both conductors
- Perfect shielding of magnetic field
- TEM modes up to a certain cut-off frequency

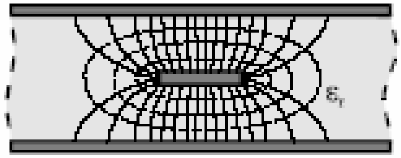
Example of transmission line: Microstip line



Triple-layer transmission line

Conductor is completely shielded between two ground planes

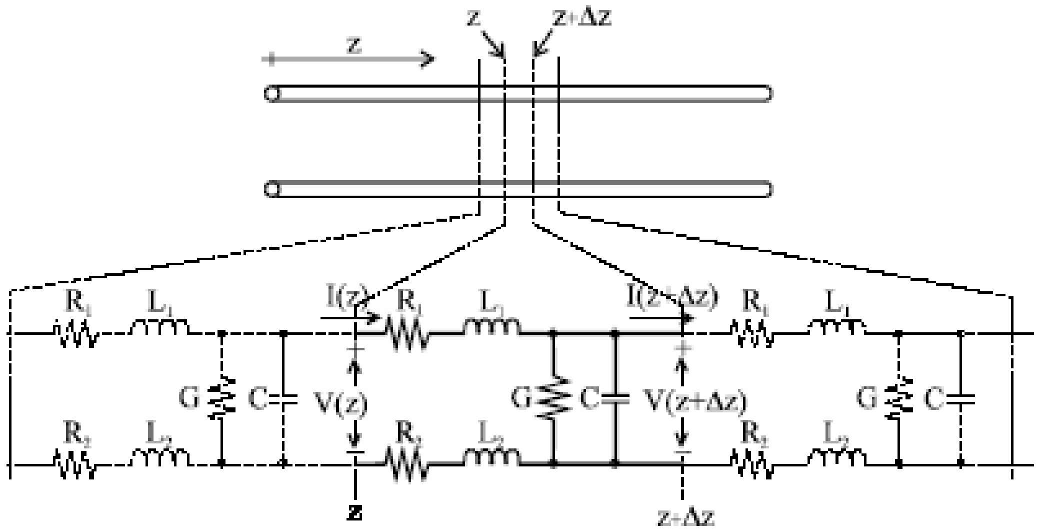




Cross-sectional view

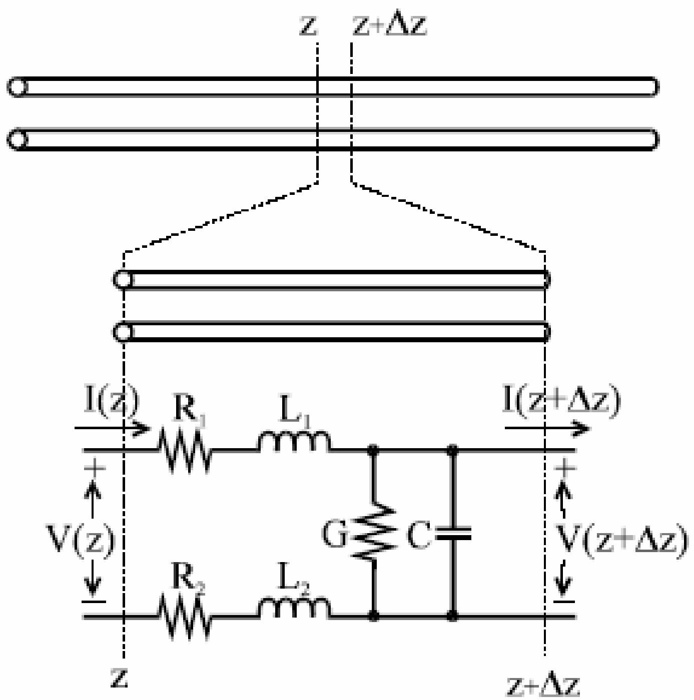
General Transmission Line Equations

- Detailed analysis of a differential section



Note: Analysis applies to all types of transmission lines such as coax cable, two-wire, microstrip, etc.

Kirchhoff's laws on a microscopic level



- Over a differential section we can again use basic circuit theory
- Model takes into account line losses and dielectric losses
- Ideal line involves only L and C

Advantages versus disadvantages of electric circuit representation

- Clear intuitive physical picture
- yields a standardized two-port network representation
- serves as building bocks to go from microscopic to macroscopic forms

- Basically a one-dimensional representation (cannot take into account interferences)
- Material nonlinearities, hysteresis, and temperature effects are not accounted for

Derivation of differential transmission line form

KVL:

$$V(z) = (R + j\omega L)I(z)\Delta z + V(z + \Delta z)$$

$$-Lim\left(\frac{V(z+\Delta z) - V(z)}{\Delta z}\right) = -\frac{dV(z)}{dz} = (R + j\omega L)I(z)$$

KCL:

$$I(z) = (G + j\omega C)\Delta z V(z + \Delta z) + I(z + \Delta z)$$

$$-\frac{dI(z)}{dz} = (G + j\omega C)V(z)$$

Coupled DE

Traveling Voltage and Current Waves

$$\frac{d^2V(z)}{dz^2} - k^2V(z) = 0 \quad \longleftrightarrow \quad \frac{d^2I(z)}{dz^2} - k^2I(z) = 0$$

where

$$k = k_r + jk_i = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Phasor expressions

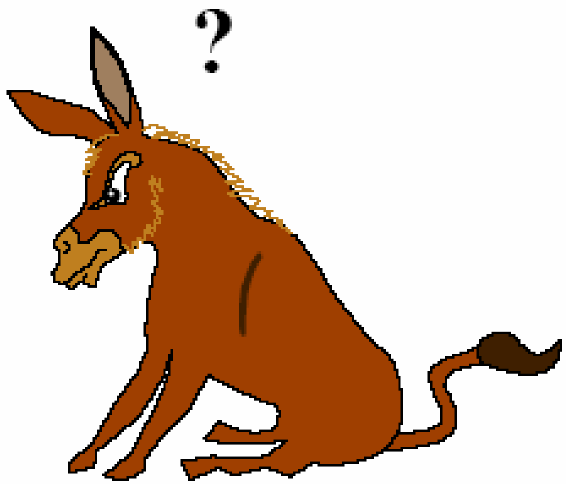
Right traveling wave

$$V(z) = V^+e^{-kz} + V^-e^{+kz} \qquad I(z) = I^+e^{-kz} + I^-e^{+kz}$$

Left traveling wave

General line impedance definition

$$I(z) = \frac{k}{(R + j\omega L)}\left(V^+ e^{-kz} - V^- e^{+kz}\right)$$



$$Z_0 = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} = \frac{V^+}{I^+} = -\frac{V^-}{I^-}$$

Characteristic line impedance

ATTENUATION, PROPAGATION CONSTANT, AND IMPEDANCE FOR VARIOUS MEDIUMS

	Any Medium	Lossless Medium ($\sigma = 0$)	Low-Loss Medium ($\varepsilon''/\varepsilon' \ll 1$)	Good Conductor ($\varepsilon''/\varepsilon' \gg 1$)	Units
α	$\omega\left\{\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}-1\right]\right\}^{1/2}$	0	$\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\pi f \mu \sigma}$	Np/m
β	$\omega\left\{\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}+1\right]\right\}^{1/2}$	$\omega\sqrt{\mu\varepsilon}$	$\omega\sqrt{\mu\varepsilon}$	$\sqrt{\pi f \mu \sigma}$	rad/m
Z_o	$\sqrt{\frac{\mu}{\varepsilon'}}\left(1-j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$(1+j)\frac{\alpha}{\sigma}$	Ω
v_p	ω/β	$\frac{1}{\sqrt{\mu\varepsilon}}$	$\frac{1}{\sqrt{\mu\varepsilon}}$	$\sqrt{\frac{4\pi f}{\mu\sigma}}$	m/s
λ	$2\pi/\beta = v_p/f$	v_p/f	v_p/f	v_p/f	m

Notes: $\varepsilon'' = \sigma/\omega$; In free space, $\varepsilon = \varepsilon_o$, $\mu = \mu_o$; In practice, a material is considered a low-loss medium if $\varepsilon''/\varepsilon' = \sigma/\omega\varepsilon < 0.01$ and a good conducting medium if $\varepsilon''/\varepsilon' = \sigma/\omega\varepsilon > 100$.