

**ELEC 412 RF**  
**& Microwave Engineering**

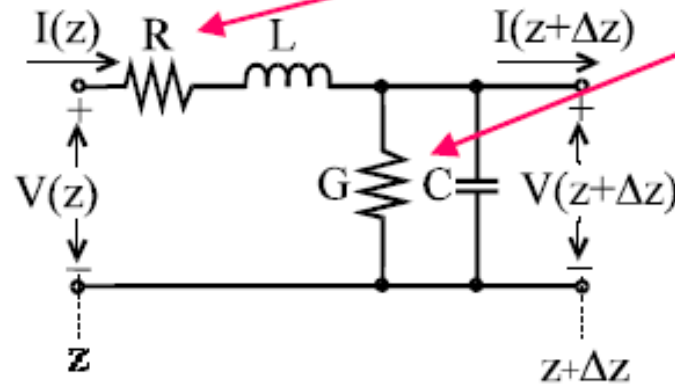
**Fall 2004**

**Lecture 3**

# Lossless Transmission Line Model

- Line representation

Lossless implies:  
 $R = G = 0!$



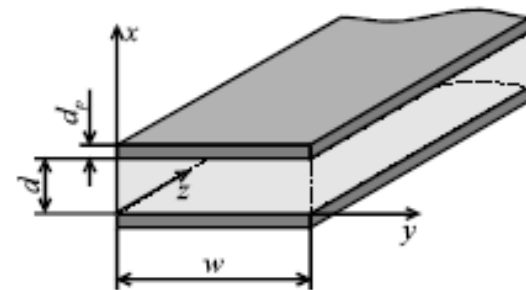
Characteristic impedance:

$$Z_0 = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}$$

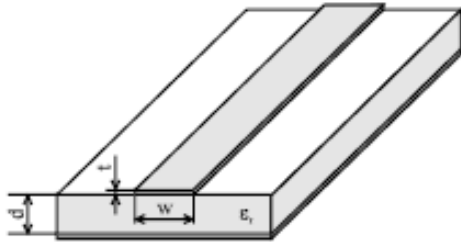
**Note:**  $R, L, G, C$  are given per unit length and depend on geometry

## Transmission Line Parameters for different line types

	2-wire	coax	parallel-plate
R	$\frac{1}{\pi a \sigma \delta}$	$\frac{1}{2\pi \sigma \delta} \left( \frac{1}{a} + \frac{1}{b} \right)$	$\frac{2}{w \sigma \delta}$
L	$\frac{\mu}{\pi} ch^{-1} \left( \frac{D}{2a} \right)$	$\frac{\mu}{2\pi} \ln \left( \frac{b}{a} \right)$	$\mu \frac{d}{w}$
G	$\frac{\pi \sigma}{ch^{-1}(D/(2a))}$	$\frac{2\pi \sigma}{\ln(b/a)}$	$\sigma \frac{w}{d}$
C	$\frac{\pi \epsilon}{ch^{-1}(D/(2a))}$	$\frac{2\pi \epsilon}{\ln(b/a)}$	$\epsilon \frac{w}{d}$

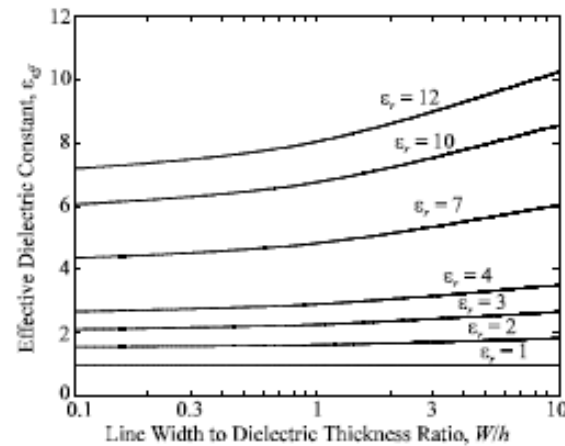
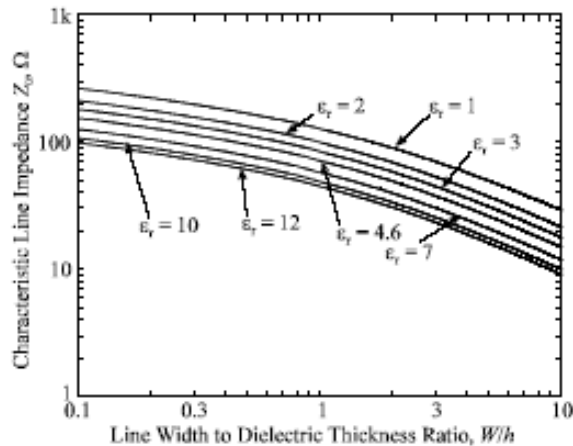


# Microstrip line

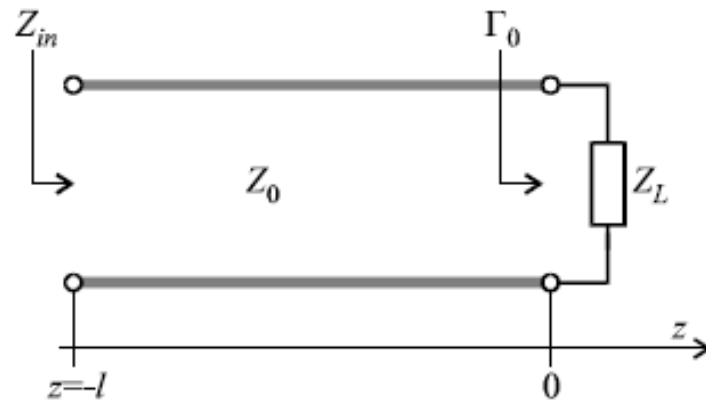


$$Z_0 = \frac{\sqrt{\mu_0/\epsilon_0}}{2\pi\sqrt{\epsilon_{eff}}} \ln\left(8\frac{h}{W} + \frac{W}{4h}\right), W/h < 1$$

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[ (1 + 12h/W)^{-1/2} + 0.04(1 - W/h)^2 \right]$$



## What is a voltage reflection coefficient?



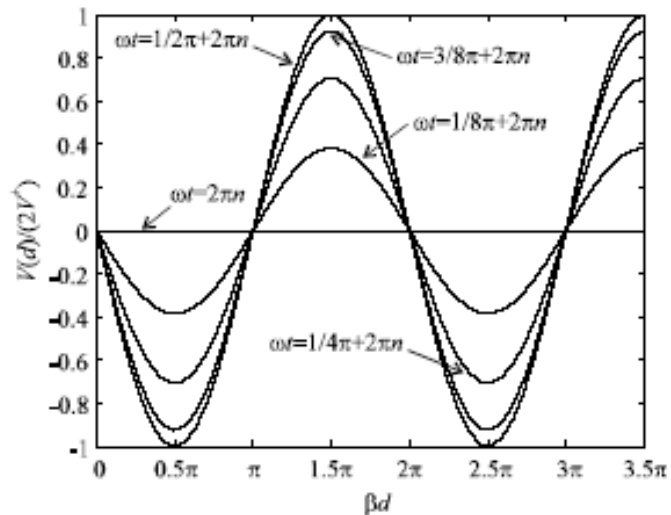
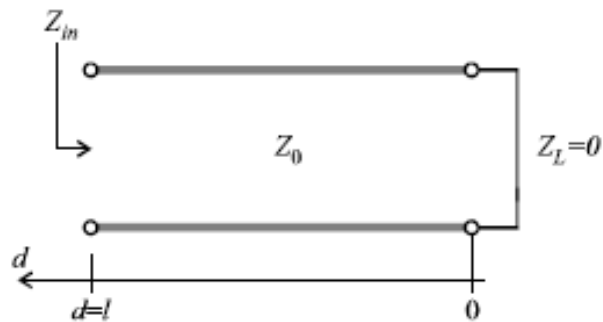
Reflection coefficient  
at the load location

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$\Gamma_0 = 1$  ( $Z_L \rightarrow \infty$ )

$\Gamma_0 = -1$  ( $Z_L \rightarrow 0$ )

## Standing Waves

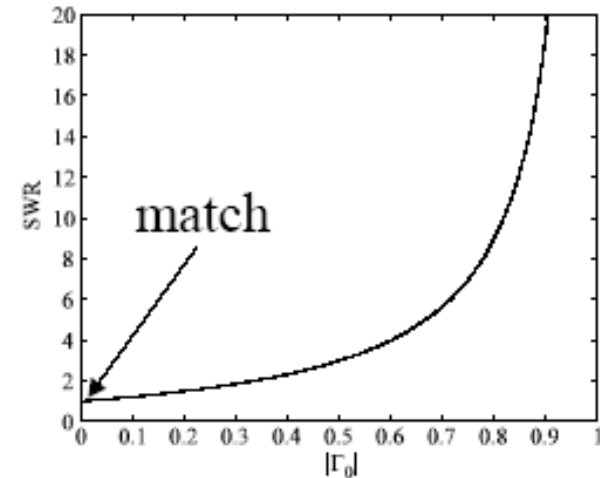


$$V(d) = V^+ (e^{+j\beta d} - e^{-j\beta d})$$

$$v(d, t) = 2V^+ \sin(\beta d) \cos(\omega t + \pi / 2)$$

## Standing wave ratio

$$SWR = \frac{|V_{\max}|}{|V_{\min}|} = \frac{|I_{\max}|}{|I_{\min}|} = \frac{1 + |\Gamma_0|}{1 - |\Gamma_0|}$$



SWR is a measure of mismatch of the load to the line

SWR=1 (matched) or SWR  $\rightarrow \infty$  (total mismatch)