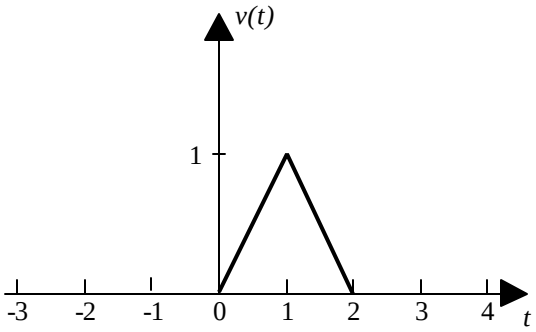
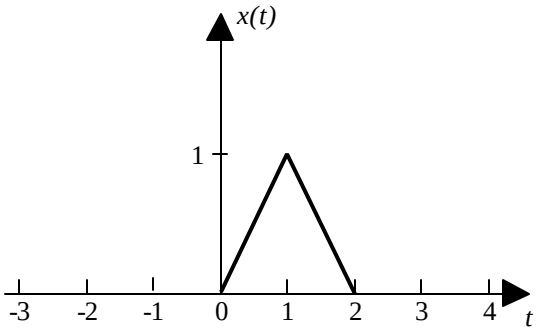
EXAMPLE # 2

Convolve the following functions:

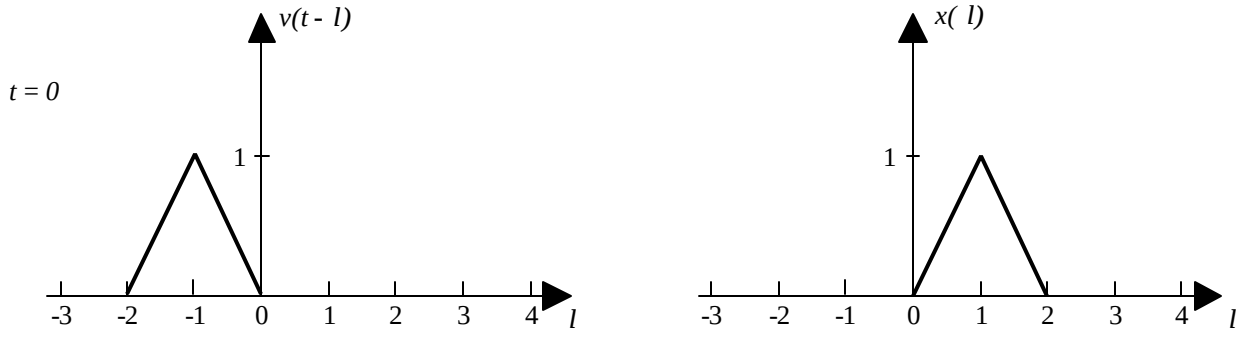
$$x(t) = \Lambda\left(\frac{t-1}{2}\right)$$

$$v(t) = \Lambda\left(\frac{t-1}{2}\right)$$





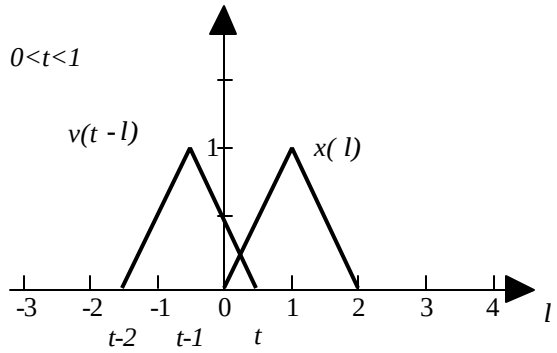
Choose to flip $v(t)$ about the y-axis. This will then be the convolution at $t = 0$.



The first region of integration is $t<0$ for which no overlapping area exists. We construct a table as follows:

Time shift , t Region of Overlap	l lower limit of integration	l upper limit of integration	Area of Overlap (Integral)
$t < 0$	---	---	0

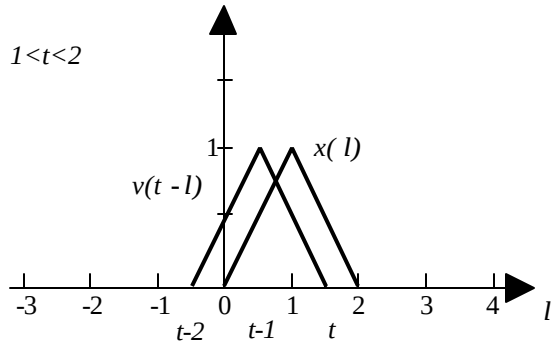
For ease of viewing the "overlap" intervals, the two functions are drawn on the same axes:



The function $v(t - l)$ slides over the function $x(l)$. The second region region of overlap and function to be integrated is $0 \le t < 1$ with the lower limit of the overlapping area determined by $x(l)$ and the upper limit t of the area is determined by $v(t - l)$.

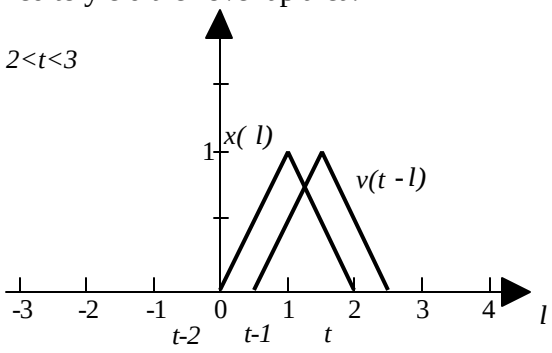
Time shift , t Region of Overlap	l lower limit of integration	l upper limit of integration	Area of Overlap (Integral)
$t < 0$	---	---	0
$0 \le t < 1$	0	t	$\int_{\lambda\, lower}^{\lambda\, upper} (t-\lambda)\lambda d\lambda$

The third region of overlap is $1 \le t < 2$ as shown below. There is complete overlap of the two areas. Three different integrals must be added to yield the "overlap area" defined by $v(t - l)$ in the region $1 \le t < 2$.



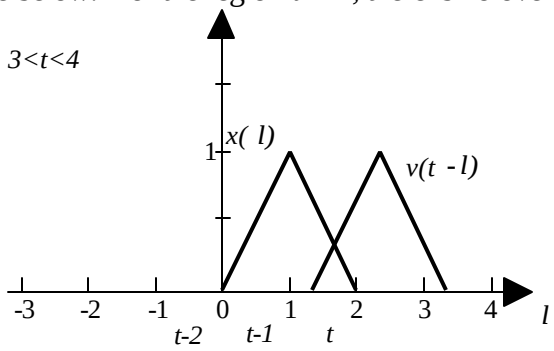
Time shift , t Region of Overlap	l lower limit of integration	l upper limit of integration	Area of Overlap (Integral)
$t < 0$	---	---	0
$0 \le t < 1$	0	t	$\int_{\lambda\,lower}^{\lambda\,upper} (t-\lambda)\lambda d\lambda$
$1 \le t < 2$	0 $t-1$ 1	$t-1$ 1 t	$\int_{\lambda\,lower}^{\lambda\,upper} \left[(\lambda - t)+2\right]\lambda d\lambda$ $+ \int_{\lambda\,lower}^{\lambda\,upper} (t-\lambda)\lambda d\lambda$ $+ \int_{\lambda\,lower}^{\lambda\,upper} (t-\lambda)(2-\lambda) d\lambda$

The fourth region of overlap is $2 \le t < 3$. There are three limits of integration defined by the three integrals that need to be summed to yield the "overlap area."



Time shift , t Region of Overlap	l lower limit of integration	l upper limit of integration	Area of Overlap (Integral)
$t < 0$	---	---	0
$0 \le t < 1$	0	t	$\int_{\lambda\, lower}^{\lambda\, upper} (t-\lambda)\lambda d\lambda$
$1 \le t < 2$	0 $t-1$ 1	$t-1$ 1 t	$\int_{\lambda\, lower}^{\lambda\, upper} \left[(\lambda - t)+2\right]\lambda d\lambda$ $+ \int_{\lambda\, lower}^{\lambda\, upper} (t-\lambda)\lambda d\lambda$ $+ \int_{\lambda\, lower}^{\lambda\, upper} (t-\lambda)(2-\lambda) d\lambda$
$2 \le t < 3$	$t-2$ 1 $t-1$	1 $t-1$ 2	$\int_{\lambda\, lower}^{\lambda\, upper} \left[(\lambda - t)+2\right]\lambda d\lambda$ $+ \int_{\lambda\, lower}^{\lambda\, upper} \left[(\lambda - t)+2\right](2-\lambda) d\lambda$ $+ \int_{\lambda\, lower}^{\lambda\, upper} (t-\lambda)(2-\lambda) d\lambda$

The next region is $3 \le t < 4$ with the lower and upper limits of $t - 2$ and 2, respectively, with the integral of the area shown in the table below. For the region $t > 4$, there is no overlap so the integral is zero.



Time shift , t Region of Overlap	l lower limit of integration	l upper limit of integration	Area of Overlap (Integral)
$t < 0$	---	---	0
$0 \le t < 1$	0	t	$\int_{\lambda\, lower}^{\lambda\, upper} (t-\lambda)\lambda d\lambda$
$1 \le t < 2$	0 $t-1$ 1	$t-1$ 1 t	$\int_{\lambda\, lower}^{\lambda\, upper} \left[(\lambda - t)+2\right]\lambda d\lambda$ $+ \int_{\lambda\, lower}^{\lambda\, upper} (t-\lambda)\lambda d\lambda$ $+ \int_{\lambda\, lower}^{\lambda\, upper} (t-\lambda)(2-\lambda) d\lambda$
$2 \le t < 3$	$t-2$ 1 $t-1$	1 $t-1$ 2	$\int_{\lambda\, lower}^{\lambda\, upper} \left[(\lambda - t)+2\right]\lambda d\lambda$ $+ \int_{\lambda\, lower}^{\lambda\, upper} \left[(\lambda - t)+2\right](2-\lambda) d\lambda$ $+ \int_{\lambda\, lower}^{\lambda\, upper} (t-\lambda)(2-\lambda) d\lambda$
$3 \le t < 4$	$t-2$	2	$\int_{\lambda\, lower}^{\lambda\, upper} \left[(\lambda - t)+2\right](2-\lambda) d\lambda$
$t > 4$	---	---	0