

Properties of Convolution

Associativity

For any signals $x(t)$, $v(t)$, and $w(t)$,

$$(x(t) * v(t)) * w(t) = x(t) * (v(t) * w(t)) .$$

Commutativity

$$x(t) * v(t) = v(t) * x(t) .$$

Distributivity with Addition

$$x(t) * (v(t) + w(t)) = x(t) * v(t) + x(t) * w(t) .$$

Shift Property

$$x(t - c) * v(t - c) = x(t - c) * v(t) = x(t) * v(t - c) .$$

Derivative Property

If the signal $x(t)$ has an ordinary first derivative $\dot{x}(t)$, the convolution $x(t) * v(t)$ has an ordinary first derivative,

$$\frac{d}{dt} [x(t) * v(t)] = \dot{x}(t) * v(t) = x(t) * \dot{v}(t) .$$

If both $x(t)$ and $v(t)$ are differentiable then,

$$\frac{d^2}{dt^2} [x(t) * v(t)] = \dot{x}(t) * \dot{v}(t) .$$

Integration Property

Let $x^{(-1)}(t)$ and $v^{(-1)}(t)$ denote integrals of $x(t)$ and $v(t)$,

$$x^{(-1)}(t) = \int_{-\infty}^t x(\lambda) d\lambda \quad \text{and} \quad v^{(-1)}(t) = \int_{-\infty}^t v(\lambda) d\lambda .$$

Then,

Convolution Properties

$$[x(t) * v(t)]^{(-1)} = x^{(-1)}(t) * v(t) = x(t) * v^{(-1)}(t) \quad .$$

Convolution with the Unit Impulse

$$x(t) * \mathbf{d}(t) = \mathbf{d}(t) * x(t) = x(t) \quad .$$

Convolution with the Shifted Unit Impulse

$$x(t) * \mathbf{d}(t - c) = x(t - c) \quad .$$