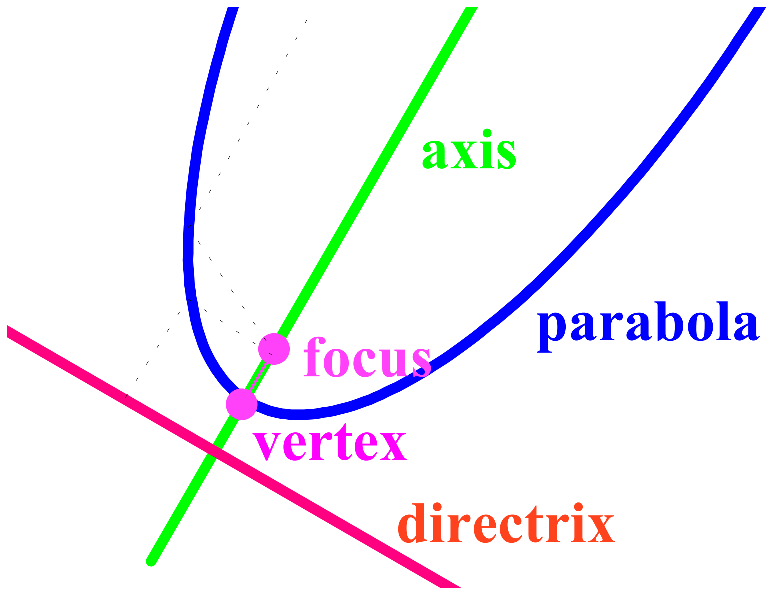
Math 140 Lecture 28

$y = ax^2 + bx + c$ is a vertical parabola. It has a vertical axis of symmetry. Other parabolas have horizontal or slanted axes.



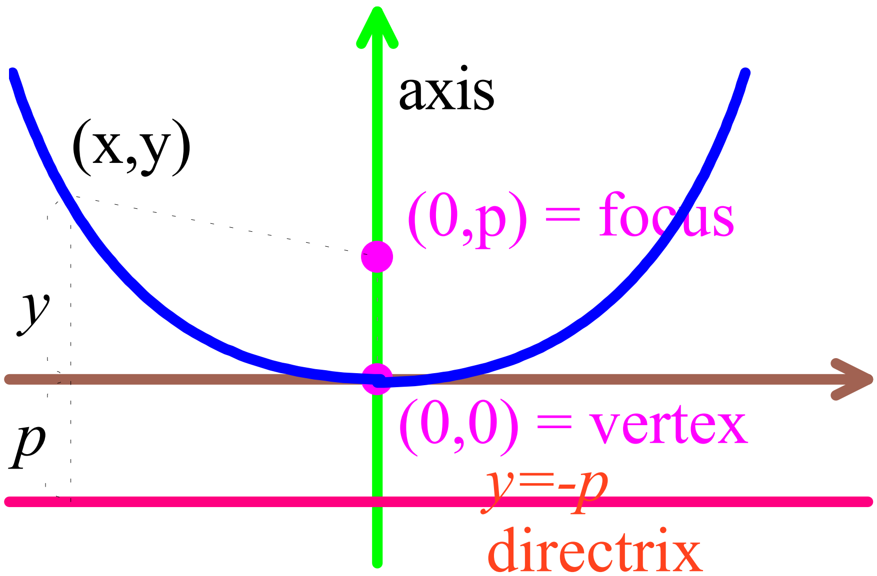
DEFINITION.

A parabola consists of all points equidistant between a given focus point and a given directrix line.

The axis is the line through the focus and perpendicular to the directrix.

The vertex is the intersection of the parabola and the axis. It lies halfway between the focus and the directrix.

- Find the equation for the parabola with focus $(0, p)$ and directrix $y = -p$.



For any point (x, y),
The distance between (x, y) and the directrix $y = -p$ is
$\boldsymbol{y+p}$.
The distance between (x, y) and the focus $(0, p)$ is

$$\sqrt{(x-0)^2+(y-p)^2} = \sqrt{x^2+y^2-2py+p^2}\ .$$

(x, y) is on the parabola iff the distances are equal

iff $y+p = \sqrt{x^2+y^2-2py+p^2}$

iff $(y+p)^2 = x^2+y^2-2py+p^2$

iff $y^2+2py+p^2 = x^2+y^2-2py+p^2$

iff $2py = x^2-2py$

iff $4py = x^2$

iff $x^2 = 4py$

iff $x^2 = ky$ where $k = 4p$ and $p = k/4$

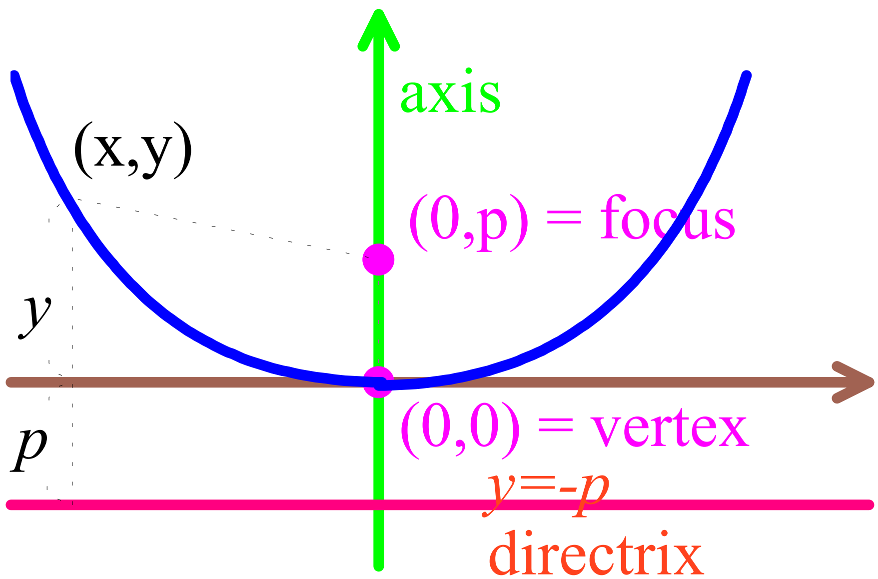
Vertical Parabola Theorem. For $k \neq 0$, $x^2 = ky$ is a vertical parabola with:

Vertex = (0,0).

Focus = (0, p).

Directrix: $y = -p$ where $p = \dfrac{k}{4}$.

Axis = the y-axis.



Exchanging x and y gives —

Horizontal Parabola Theorem. For $k \neq 0$,

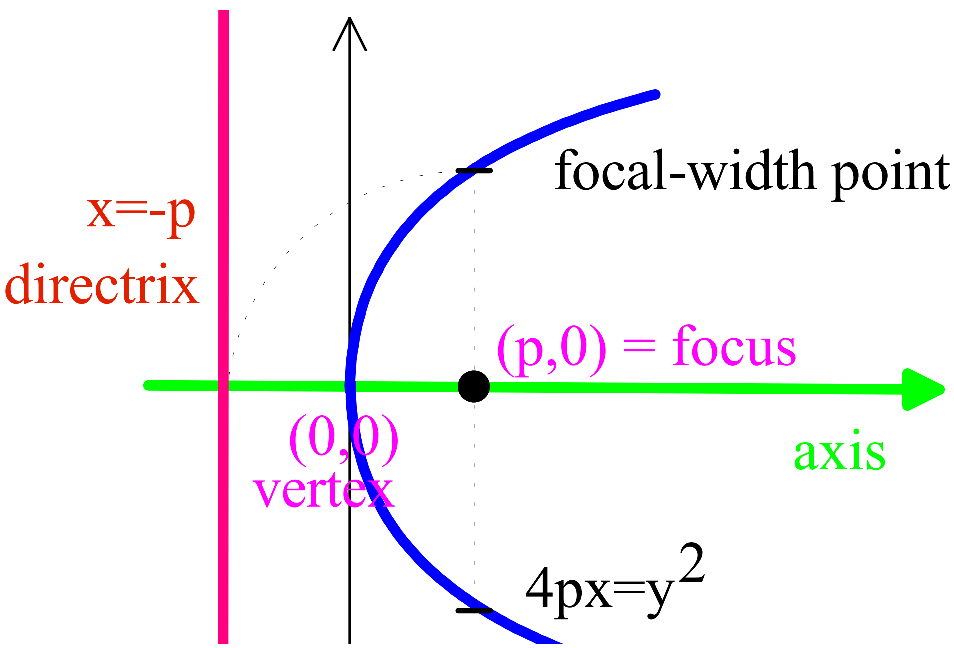
$y^2 = kx$ is a horizontal parabola with:

Vertex = (0, 0).

Focus = (p, 0).

Directrix: $x = -p$ where $p = \frac{k}{4}$

Axis = the x-axis;



x^2-parabolas like $y = x^2$ are vertical;

y^2-parabolas are horizontal.

- Find the focus, directrix and graph of $y = -x^2/8$.
- Find the focus, directrix and graph of $3y^2 = 4x$.

Theorem. In any equation,

Replacing each		shifts the graph
x by $x-a$	$\rightarrow$	*right* a units
x by $x+a$	$\leftarrow$	*left* a units
y by $y-b$	$\uparrow$	*up* b units
y by $y+b$	$\downarrow$	*down* b units

Shift both the parabola and its focus, directrix, vertex.

To graph a parabola,

- get the squared variable on the left, the rest on the right.
- Complete the square if needed.
- Write the equation in
  vertical parabola form: $(x \pm a)^2 = k(y \pm b)$ or
  horizontal parabola form: $(y \pm b)^2 = k(x \pm a)$

- Find the focus, directrix and graph:
  $$x^2 - 2x + 9 - 8y = 0$$
- Find the focus, directrix and graph:
  $$y^2 + 2y = 4x - 5.$$

□