Children's Development of Meaningful Fraction Algorithms: A Kid's Cookies and a Puppy's Pills

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Many children have difficulty developing connected knowledge about fractions concerning number sense, operation sense, and algorithmic skills (Behr et al., 1992; Kieren, 1988). Procedural knowledge, such as algorithms for operations, is often taught without context or concepts, implying to the learner that algorithms are an ungrounded code only mastered through memorization. Introducing algorithms before conceptual understanding is established, or without linking the algorithm to conceptual knowledge, creates a curriculum that tends to be perplexing for children to master or appreciate (Carpenter, 1986). This untimely rush toward symbol manipulation fosters misconceptions about a lack of connectedness both between concepts and procedures and between fractions and students' everyday lives. However, once children have developed a conceptual knowledge base for fraction sense and operation sense, they can meaningfully learn, or even create for themselves, appropriate fraction algorithms. Kieren (1988) hypothesized that children gradually expand their knowledge and thinking about fractions by building it up from personal environments. As children learn, they develop more intuitive knowledge in which they combine thought, informal language, and images. They become more able to extract mentally, and think about, fractional ideas without a strong dependence on the specific context. Eventually, they begin using formal symbols when they become able to connect concepts and procedures and to use and understand conventional language, notations, and algorithms.

Two Children Develop Personal Algorithms

We shall describe two girls who developed personal algorithms, based on conceptual knowledge for fractions. First, you will meet Joe and his daughter, Leah. Leah used sophisticated mental strategies to divide whole numbers by whole numbers and add fractions with different denominators. Second, you will meet Janet and her student, Stephanie. Stephanie used explicit pictorial and symbolic strategies to divide whole numbers by fractions. Both girls developed their strategies through encounters with real-world division situations and with encouragement from adults.

Kids and Cookies

When Leah was almost five years old, Leah and Joe often played games while driving to Leah's preschool in their pickup truck. In one game, which Leah named "Kids and Cookies," Joe posed situations and questions to help Leah establish conceptual understanding of division and fractions. He would give her a number of kids and a number of cookies, and ask Leah how she would share the cookies. At first Joe gave Leah specific, real-to-Leah situations that led to whole-number responses (i.e., What if Molly and Lee were at our house and we had 6 cookies, how would you share the cookies?), and then later he presented her with less descriptive and less personal situations that led to fractional results. Below is an excerpt of a conversation that took place one morning (reconstructed after Joe arrived at his office):

Joe: Hey Leah, what do you want to do today?
Leah: Let's play Kids and Cookies.
Joe: OK. What if you had 4 cookies and 3 kids? How would you share them?
Leah: One, one, and one, and then there is one left. Then they each get one third, one third, one third.
Joe: So, how much does each kid get?
Leah: They get one whole one and one third.
Joe: What if you had 5 cookies and 3 kids? How could you share the cookies?
Leah: One, one, one. Then there's two more left. OK. Then, they get a third, a third, a third and then a third, a third, a third.
Can they share them?

Here is a table to compare:

<table>
<thead>
<tr>
<th>Item</th>
<th>Quantity</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apple</td>
<td>5</td>
<td>Fresh</td>
</tr>
<tr>
<td>Orange</td>
<td>3</td>
<td>Dried</td>
</tr>
<tr>
<td>Banana</td>
<td>2</td>
<td>Ripe</td>
</tr>
<tr>
<td>Carrot</td>
<td>4</td>
<td>Stale</td>
</tr>
</tbody>
</table>

The table shows the quantity, type, and status of each item. For example, the apple has 5, is fresh, and the orange has 3, is dried.
Recommended Teaching Practices

1. Convergent/Converging Concept to Develop Concepts

- Introduce the concept of fractions as parts of a whole.
- Use visual aids such as fraction circles or bars to help students understand the concept.
- Relate the concept to real-life situations, such as sharing a pizza or a cake.

2. Convergent/Diverging Concept to Develop Concepts

- Teach the concept of fractions as a ratio of two quantities.
- Use manipulatives such as fraction tiles or blocks to help students visualize the concept.
- Relate the concept to real-life situations, such as comparing quantities.

3. Convergent/Converging Concept to Develop Procedures

- Introduce the concept of procedures for finding equivalent fractions.
- Use visual aids such as fraction bars or number lines to help students understand the concept.
- Relate the procedure to real-life situations, such as comparing quantities.

4. Convergent/Diverging Concept to Develop Procedures

- Teach the concept of procedures for simplifying fractions.
- Use manipulatives such as fraction tiles or blocks to help students visualize the concept.
- Relate the procedure to real-life situations, such as comparing quantities.

5. Convergent/Converging Concept to Develop Algorithms

- Introduce the concept of algorithms for adding and subtracting fractions.
- Use visual aids such as fraction bars or number lines to help students understand the concept.
- Relate the algorithm to real-life situations, such as comparing quantities.

6. Convergent/Diverging Concept to Develop Algorithms

- Teach the concept of algorithms for multiplying and dividing fractions.
- Use manipulatives such as fraction tiles or blocks to help students visualize the concept.
- Relate the algorithm to real-life situations, such as comparing quantities.

Final Comment

- Encourage children to keep track of their progress and reflect on their learning.
- Provide opportunities for students to share their understanding of fractions with each other.
- Use a variety of teaching strategies to cater to different learning styles.
- Encourage students to use fraction notation and symbols appropriately.

- Conclude with a review of key concepts and procedures.
- Encourage students to ask questions and seek clarification as needed.
- Provide opportunities for students to reflect on their learning and set goals for future study.

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