Teaching of Mathematics in Singapore Schools
Berinderjeet Kaur
National Institute of Education
Singapore

Abstract
In Singapore, Mathematics is a core subject in the school curriculum for both the primary and secondary schools. The primary aim of the mathematics curriculum is to enable pupils to develop their ability in mathematical problem solving. The conceptualization of the mathematics syllabuses is based on a framework that emphasizes the interplay of five components – Concepts, Skills, Processes, Attitudes and Metacognition to achieve this aim.
Three initiatives were launched in Singapore’s education system in 1997. They are National Education, Information Technology (IT) and Critical and Creative Thinking. With the infusion of these initiatives in all curriculum subjects at schools, the teaching of mathematics has been revolutionized. During mathematics lessons examples are drawn from prevailing national and current issues to reinforce pupils’ understanding of their nation’s constraints and challenges. IT is used whenever its use enhances the learning of mathematics and thinking skills are means by which teachers develop self-regulated learners and problem solvers. To forge the vision: THINKING SCHOOLS, LEARNING NATION ahead, teachers have been identified as the key and hence their development has been accorded due importance. Over the past couple of years, mathematics teachers have been focused on excellence in their mathematics classrooms.

Introduction
In the history of Singapore’s education landscape, the period 1996 till the present could best be described as “The Way Forward...” (Kaur, 2003a). On 8th September 1996, Prime Minister Goh Chok Tong’s Teachers’ Day Rally Speech (Goh, 1996) marked the beginning of a slew of initiatives that have made a significant impact on all classrooms in Singapore schools. Three initiatives were launched in Singapore’s education system in 1997 (Ministry of Education, 1998). They are National Education, Information Technology (IT) and Critical and Creative Thinking. The vision, THINKING SCHOOLS, LEARNING NATION (TSLN) was also unveiled by Prime Minister Goh Chok Tong at the opening of the Seventh International Conference on Thinking held in Singapore, in June 1997 (Goh, 1997). To forge the TSLN vision ahead, teachers have been identified as the key and hence their professional development and performance has been accorded significant importance (Ministry of Education, 2000a).

School Mathematics Curriculum
In Singapore, mathematics is a core subject in the school curriculum for both the primary and secondary schools. The primary aim of the mathematics curriculum is to enable pupils to develop their ability in mathematical problem solving. The conceptualization of the mathematics syllabuses is based on a framework, shown in figure 1 (Ministry of Education, 2000b; 2000c).
In Singapore, although the study of mathematics is compulsory both in the primary and secondary schools the breadth and depth of mathematics taught to pupils depend on their ability (Kaur, 2003b). Core mathematical concepts are common to all courses and the depth to which the topics are taught at a particular year level differs among the courses. The school mathematics curriculum has mathematical problem solving as its primary goal. The attainment of this mathematical ability is dependent on five inter-related components – Concepts, Skills, Processes, Attitudes and Metacognition.

Concepts refer to the basic mathematical knowledge (numerical, geometrical, algebraic and statistical concepts) needed for solving mathematical problems. Skills refer to the topic-related manipulative skills (estimation and approximation, mental calculation, communication, use of mathematical tools, arithmetic manipulation, algebraic manipulation and handling data) that pupils are expected to use when solving problems. Processes refer to the thinking skills (classifying, comparing, identifying attributes and components, sequencing, induction, deduction, generalizing, verifying and spatial visualization) and heuristics (act it out, use a diagram/model, use guess and check, make a systematic list, look for patterns, work backwards, use before-after concept, make suppositions, restate the problem in another way, simplify the problem, solve part of the problem, think of a related problem and use equations). Attitudes refer to the affective aspects of mathematics learning such as: enjoying doing mathematics, showing confidence in using mathematics, persevering in solving a problem, and appreciating the beauty and power of mathematics. Metacognition refers to the ability to control one’s own thinking processes in problem solving. This includes: constant and conscious monitoring of the strategies and thinking processes used in carrying out a task; seeking alternative ways of performing a task; and checking appropriateness and reasonableness of answers.
The mathematics syllabuses in Singapore, for schools, are issued by the Ministry of Education in collaboration with University of Cambridge Local Examinations Syndicate. Every ten years or so, the mathematics syllabuses undergo a periodic review to ensure that they remain relevant so as to prepare pupils for the challenges and opportunities of the future and also to be in line with the national objectives. The syllabuses adopt a spiral approach. The syllabuses are a guide for teachers to plan their mathematics instructional program. Teachers are not bound by the sequence of topics but ensure that the hierarchy and linkage are maintained. Teachers are free to exercise flexibility and creativity when drawing up plans of work which serve as a blueprint for them to implement the instructional programme.

The Three Initiatives

National Education

During Teachers’ Day Rally on 8th September 1996 (Goh, 1996), Singapore’s Prime Minister, Mr Goh Chok Tong announced the need to strengthen National Education in the education system. In 1997, the National Education initiative was launched by Deputy Prime Minister BG Lee Hsien Loong (Lee, 1997). The objective of National Education is to develop national cohesion, the instinct for survival and confidence in the future, by fostering a sense of identity, pride and self-respect as Singaporeans. This is to be achieved by knowing the Singapore story - how Singapore succeeded against the odds to become a nation, by understanding Singapore’s unique challenges, constraints and vulnerabilities, which make Singapore different from other countries and by instilling the nation’s core values in the young so as to ensure Singapore’s continued success and well-being.

National Education is not a formal subject on its own but is infused across subjects in the curriculum. Six National Education messages have guided the National Education effort over the years. The six messages are:

- Singapore is our homeland. This is where we belong. We want to keep our heritage and way of life.
- We must preserve racial and religious harmony. Though many races, religions, languages and cultures, we pursue one destiny.
- We must uphold meritocracy and prevent corruption. This provides opportunity for all according to their ability and effort.
- No one owes Singapore a living. We must find our own way to survive and prosper.
- We must ourselves defend Singapore. No one else is responsible for our security and well-being.
- We have confidence in the future. United, determined, and well prepared, we shall build a bright future for ourselves.

These six messages can be seen as a set of desired outcomes – the conclusions that we want the pupils to come to at the end of any programme or activity.

When National Education was first launched, many mathematics teachers found it virtually impossible to infuse it in their lessons. This was mainly due to their perception that National Education was all about the History of Singapore. In 1999, at a workshop for teachers on National Education in the mathematics classroom, I defined National Education as follows:

Sensitizing pupils to past and present national issues by using examples during our lessons – depending on how these can be infused into the topics we are teaching.
Today, mathematics teachers often draw on examples from the prevailing national and global issues to provide pupils’ opportunities to understand the nation’s constraints and challenges.

Wong (2003), summarized the different ways by which National Education could be infused in mathematics lessons using a framework that is a matrix, shown in Figure 2.

<table>
<thead>
<tr>
<th>National Education</th>
<th>Mathematics Education</th>
</tr>
</thead>
<tbody>
<tr>
<td>Environments</td>
<td>Contents A, Processes B, Culture C</td>
</tr>
<tr>
<td>Values</td>
<td>Contents D, Processes E, Culture F</td>
</tr>
</tbody>
</table>

Figure 2: Matrix of Mathematics-based National Education

The two components under the National Education dimension are the Singapore environments and espoused values. The Environments component deals with various aspects of Singapore’s geographical, historical, social, cultural, political, and economic environments. These environments cover past and current events and various campaigns held at school, local and national levels. The espoused national values – united regardless of race, language or religion, justice, equality, democracy, to be the best one can be, sound morals and cultural heritage, compassion, kindness, open-mindedness and to care and share.

The three components under Mathematics Education dimension cover aspects of the mathematics curriculum. Contents represent the collection of facts and skills to be mastered. Processes refer to reasoning and making sense of the world through numbers, measurements and symbols. Mathematical processes must be applied to content and hence the two components are inter-related. They are shown separately in the matrix for the purpose of emphasis of particular activities. The third component, Culture refers loosely to things about mathematics, e.g. life stories of mathematicians, nature of their work, etc.

The six cells of the matrix labeled A to F indicate potential interactions between National Education and Mathematics Education. Examples of mathematics lessons with National Education infused are:

A – Pupils to develop their numerical measures by making estimates of the longest bridge in Singapore, percentage of population living in particular types of dwellings.
B – Pupils to find relationship between population and area of specific localities in Singapore.
C – Pupils to study the various kinds of calendars used by Singaporeans of different ethnic and religious groups.
D & E – Pupils to locate and study data on the welfare of the aged and sources of funding for old homes.
F – Pupils to read about the life history of mathematician/s and appreciate their struggles in life.

Information Technology
To keep abreast with global developments in the field of technology, the Ministry of Education’s Masterplan for Information Technology (IT) was launched on 28th April 1997 (Teo, 1997). This initiative in schools was the quickest and surest way to realize Singapore’s IT2000 Masterplan to transform the country into an intelligent island. Like all new initiatives, needless to say there were many teething problems and pedagogical issues. With all the support for infrastructure and teacher training from the Ministry of Education we have now come a long way.

Today, every classroom is equipped with projection systems that allow the teacher to use a computer for his or her teaching. Every school has computer laboratories for pupils to do IT based lessons. IT is now seamlessly integrated in the teaching of Mathematics. It is used wherever and whenever its use enhances the learning of mathematics. Other than tools such as Microsoft Powerpoint (Microsoft, 2003a) and Microsoft Excel (Microsoft, 2003b) teachers often use specific IT tools for mathematics such as:

a) Graphmatica (Hertzer, 1991)
Graphmatica is a friendly equation plotter written by Keith Hertzer, Ksoft, Inc. It can be downloaded from the following website: http://www8.pair.com/ksoft. Graphmatica is an easy-to-use, powerful equation plotter with numerical and calculus features. Graph Cartesian functions, relations, and inequalities, plus polar, parametric, and ordinary differential equations……In summary, a great tool for students and teachers of anything from high-school algebra through college calculus.

b) The Geometer’s Sketchpad (GSP) (Key Curriculum Press, 1995)
The Geometer’s Sketchpad (or GSP for short) is a tool for drawing / constructing geometrical figures. It comprises of menus and toolbox. For constructions, geometrical instruments such as: straight edge and compasses are used. It also has a graph menu and a calculator.

c) Dynamic Mathematics Series CD Roms
The Dynamic Mathematics Series is produced by Curriculum Development Institute of Singapore (CDIS) and Educational Technology Division (ETD) of the Ministry of Education. This series comprises of 6 CD-Roms. They are:
- Building a City with Linear Graphs (CDIS, 1994a);
- Space Trek Through Symmetry (CDIS, 1994b);
- The Undersea World of Algebra (CDIS, 1995a);
- Through the Ages with Congruency and Similarity (CDIS, 1995b);
- Jungle Survival with Quadratic Equations (CDIS, 1996); and
- The Business of Graphs (ETD, 1997).

d) The Internet
The internet is a rich resource for mathematics lessons. Teachers often use appropriate web-pages and applets to demonstrate proofs such as that of Pythagoras Theorem, visualization of displacement and velocity graphs, etc.

Some Mathematics teachers also use more advanced tools such as Computer Algebra Systems. Many schools have also embarked on Learning Management Systems for Mathematics and engage pupils in on-line Mathematics learning.
Thinking Skills Initiative
In 1997, the 7th International Conference on Thinking was held in Singapore. The Prime Minister, Mr Goh Chok Tong, in his opening address (Goh, 1997) at the conference noted that Singapore has a strong education system, one that is widely recognized for having produced high levels of achievements among pupils of all abilities. However, he also cautioned that what may have worked well in the past will not work well for the future as the old formulae for success are unlikely to prepare young Singaporeans for the new circumstances and new problems they will face in the new millennium. He emphasized that we must ensure our young can think for themselves, so that the next and future generations can find their own solutions to whatever problems they may encounter. He also announced at the opening of the conference that Singapore’s vision for meeting this challenge is encapsulated in four words: “Thinking Schools, Learning Nation”. Hence the launch of the Thinking Programme in all schools.

The aims of the Thinking Programme are to enable our students to:

a) acquire and understand core thinking skills and the processes involved in using them;
b) apply these skills in the learning of content subjects and in real-life decision making and problem-solving situations; and
c) develop positive habits which would help them become critical, creative and self-regulated learners.

Our Thinking Programme adopts a two-prong approach to teaching thinking:

a) the explicit teaching of thinking skills, where the following eight core skills are taught
   - Information Gathering,
   - Remembering,
   - Focusing,
   - Organizing,
   - Analyzing,
   - Evaluating,
   - Generating, and
   - Integrating.

b) the infusion of the above skills into the core subjects like Mathematics.
In addition to the above generic skills, thinking skills specific to Mathematics that are commonly used in Mathematics lessons are:

- classifying,
- comparing,
- identifying attributes and components,
- sequencing,
- induction,
- deduction,
- generalizing,
- verifying, and
- spatial visualization

In 1998, to support the implementation of the Thinking Programme, the Curriculum Planning and Development Division (CPDD) of the Ministry of Education in Singapore produced teacher support materials in the form of Lesson Plans. Two resource packages were produced for Mathematics teachers, one each for grades 7 and 8 (CPDD, 1998a; 1998b). Some examples of the use of thinking skills in the Mathematics lessons are as follows:

Pupils use the skill of

- classifying to learn about the different types of quadrilaterals,
- comparing to learn the difference between prime and composite numbers,
- induction to generalize a number pattern and derive a general formula,
- analyzing to analyze word problems, and
- verifying to verify Pythagoras Theorem, trigonometrical identities and general rules, such as the Sine and Cosine Rules.

It is common for teachers to integrate the initiatives of IT and Thinking Skills in a lesson during mathematics. One example of such an attempt is as follows:

<table>
<thead>
<tr>
<th>Use Graphmatica to observe the graph of the following function:</th>
</tr>
</thead>
<tbody>
<tr>
<td>y = \sin x</td>
</tr>
<tr>
<td>What if the function is:</td>
</tr>
<tr>
<td>y = \sin 2x</td>
</tr>
<tr>
<td>y = \sin 3x</td>
</tr>
<tr>
<td>y = \sin 5x</td>
</tr>
<tr>
<td>y = \sin \frac{1}{2} x</td>
</tr>
<tr>
<td>y = \sin \frac{1}{3} x</td>
</tr>
<tr>
<td>y = \sin \frac{1}{4} x</td>
</tr>
<tr>
<td>What have you observed about the above family of graphs?</td>
</tr>
<tr>
<td>Sketch the graph of y = a \sin x, where a = 2.</td>
</tr>
<tr>
<td>What if a = \frac{1}{4}, \frac{3}{4}, 3, 5, 10?</td>
</tr>
<tr>
<td>What have you observed about the family of graphs of y = a \sin x</td>
</tr>
<tr>
<td>Predict the graph of y = \frac{1}{4} \sin 3x.</td>
</tr>
<tr>
<td>Use Graphmatica to check your prediction.</td>
</tr>
</tbody>
</table>
Thinking Schools, Learning Nation – Teacher the key
In 1997, when the Thinking Schools, Learning Nation (TSLN) vision was unveiled it was also realized that teachers are the key to the success of the mission. From the year, 1998 onwards all teachers are entitled to 100 hours of training and core-upgrading courses each year to keep abreast with the current knowledge and skills. Over the past couple of years, mathematics teachers have been focused on excellence in their mathematics classrooms.

Professional Development of Mathematics Teachers
Professional upgrading of mathematics teachers is an on-going process in Singapore. As teachers are entitled to 100 hours of funded training each year they seek training and development from several sources. The Ministry of Education and Teachers Network continually organizes workshops, in-service courses and seminars to upgrade teachers’ knowledge and skills, to equip teachers with effective teaching strategies, and to keep teachers abreast of recent developments in mathematics education. Teachers are encouraged to engage in lifelong learning and at the school and national level many initiatives are in place to encourage the sharing of teaching ideas and good practices. The National Institute of Education is actively engaged in the constant upgrading of mathematics teachers via Diploma level and Master Degree level courses. The Association of Mathematics Educators and the Singapore Mathematical Society too play an active role in the professional development of mathematics teachers in Singapore.

Towards excellence in the mathematics classroom
From the findings of two studies with pupils in Singapore schools (Kaur, 1997 & Kaur et al., 1999) and another with mathematics teachers in Singapore (Kaur, 2004) about good mathematics teachers, it was found that many of the findings were congruent with that of the Standards for excellence in teaching mathematics in Australian schools (AAMT, 2002). Pupils and teachers in the studies concurred that a good mathematics teacher is:
- patient, helpful, humorous, hardworking, enthusiastic, systematic/organized, dedicated and courteous/pleasant;
- caring/kind, understanding, approachable, encouraging/motivating/inspiring, strict, firm and friendly;
- good in mathematics, has a sound knowledge of how pupils learn, is able to arouse and sustain interest [not boring / interesting], engages pupils through a repertoire of teaching strategies [simple and easy exposition, explains clearly, ensures pupils understand, provides individual help] and provides timely and purposeful feedback to pupils and parents.

As the saying goes – good, better, best; never let it rest till good is better and better is BEST!, generally mathematics teachers in Singapore schools are poised to do the best they can for their pupils. The commendable performance of their pupils in international studies, such as TIMSS and TIMSS-R has rejuvenated many of them.

Voices of Heads of Mathematics Departments from 8 Secondary Schools
As part of the Diploma in Departmental Management (DDM) Programme at the National Institute of Education in April 2004, 8 heads of mathematics departments (HOD) from
secondary schools were asked to share their current classroom practices during mathematics lessons with the participants of ICME-10. During the conference an audio-video presentation prepared by the HODs and myself was shown to the participants who attended my session. The eight HODs are: Ow Yong Kok Wing Calvin - Whitley Secondary School; Tay Hwee Ping - Manjusi Secondary School; Lua Bee Eng - Canberra Secondary School; Wong Ker Sin - CHIJ St Theresa’s Secondary School; Fauziah Bte Ahmad - Westwood Secondary School; Yew Chiew Yong Gina - Compassvale Secondary School; Teo Mei Lin Yvonne - Bukit View Secondary School; Wong Lai Fong - Anderson Secondary School; Ang Sock Kiang - Pasir Ris Secondary School; Toh Gek Khiaw - Northbrooks Secondary School; Lee Han Seng - Tanglin Secondary School and S Rajasekaran - Dunearn Secondary School. Excerpts of their presentation follows.

Teaching Strategies
In the teaching of mathematics, teachers have gone beyond merely teaching facts, skills and conceptual structures. Our teachers use a repertoire of teaching approaches as there is no single suitable method to teach mathematics. We emphasize mastery of knowledge and skills, critical and creative thinking, communication and problem solving as our goal. In the mathematics classroom, there is a high-quality teacher-student discourse and teachers adopt a variety of teaching approaches to engage pupils in learning.

Some of the common combinations of approaches are:

- teacher exposition followed by student practice or seatwork and homework
- investigational activity followed by conjecturing or whole class discussion
  - pupils may work independently, in pairs or in groups,
  - pupils may use technology or manipulatives to carry out their investigations.
- practical activity followed by whole class discussion
  - pupils could work independently, in pairs or in groups,
  - pupils may play mathematical games or do outdoor activities.
- problem solving or mathematical modelling followed by presentation of work by pupils in groups.
- independent learning during e-learning week. Pupils do on-line tutorials at home and submit work to teacher electronically.

The Classroom Culture
Teachers have very high expectations for their pupils and themselves in the classroom with respect to the teaching and learning of Mathematics. Some of these expectations are:

- Readiness for lesson
Pupils are expected to be ready for lesson the moment the teacher steps into the classroom. Textbooks, exercise books and any materials as required by the teacher are to be placed on their desks. Action is usually taken on persistent pupils who always forget to bring their learning materials, the purpose, which is to teach and instill self-responsibility in the pupils. Where a classroom is untidy or dirty, the teachers will also expect the pupils to clean it up very quickly before the lesson begins. This is to help set the right tone for learning.
- Homework to be handed in punctually for grading
  Pupils are also required to submit their homework (set during the previous class) on time. Usually, the teacher begins the lesson by reviewing the homework that has been set during the previous class. The teacher will also discuss any difficulties that the pupils have. Generally, teachers are very strict with deadlines for the submission of homework. Teachers are also expected to be prompt in the marking of homework, so that timely feedback is given, and misconceptions clarified.

- Corrections
  Pupils have to do corrections for solutions graded incorrect by the teacher. At times re-corrections are also done by pupils. Teachers feel that corrections are essential as pupils would then be able to understand where and why they went wrong in their earlier attempts.

- No disruption during lesson time
  When teachers are transmitting lesson-relevant information (teaching of concepts, ideas, solution strategies or methods), pupils are expected to be listening intently in their assigned seats. When it comes to teacher demonstration (illustration of procedures and demonstration of solution steps), pupils are expected to be listening as well as taking notes of the procedures and solution steps demonstrated. In both cases, teachers constantly check and monitor pupil understanding by applying different questioning techniques. Pupils are also strongly encouraged to stop the teacher at any point during the lesson to ask questions to clear their doubts.

- Lessons are focused and pupils are on task
  When pupils are assigned tasks to work independently, either alone or with their peers, the teacher would move amongst the pupils to help those with any difficulties. This is also one way of ensuring that all pupils are on-task. Pupils are expected to put down their solutions on either an exercise book or writing paper. After an apportioned time, the teacher would either discuss the solutions with the pupils orally, or get some pupils to present their solutions on the whiteboard for class discussion.

**Homework**

It may be said that in Singapore, no mathematics lesson is complete without homework assigned. Teachers treat homework as an extension of their lesson outside class time. We feel homework is necessary as Mathematics is a subject that requires constant practice. As much of the class time is used for instruction by the teacher, pupils do their practice of applying and consolidating what they have learnt in class at home.

Furthermore, homework also develops students’ personal responsibility and good time management, ensure positive reinforcement and strengthen their discipline in learning among other skills. However, we are also mindful that homework must not stress our pupils out or kill their joy and interest of learning mathematics.

**Assessment and Standards**
Assessment is an integral part of the teaching and learning of mathematics in Singapore schools. Both formative and summative assessments are carried out. Continuous assessment which is formative in nature provides both the teacher and pupils with valuable feedback on teaching strategies and pupil misconceptions. In some schools this is left entirely to the discretion of the subject teachers while in others common tests are carried out across the year levels.

Semestral assessment which is summative in nature measures pupil achievement at the end of each semester. In all schools this is a formal examination and all pupils in the same year level take the same examination paper at the same time. A school year is made up of two semesters. Pupils take a national examination at the end of their four years of secondary schooling. Throughout the four years of schooling, pupils are slowly but gradually given questions that match the standard of the national examination. Both continual and semestral assessments are rigorous and taken very seriously by both the teachers and their pupils.

**TIMSS and TIMSS-R**

Finally, we feel that the good performance of Singapore’s pupils in both the Third International Mathematics and Science Study (TIMSS) and Repeat of the Third International Mathematics and Science Study (TIMSS-R) may be attributed to:

- a sound mathematics curriculum that caters for the differing needs of the pupils,
- the streaming of pupils according to ability,
- well resourced and managed schools,
- school leaders with a common vision – Thinking Schools, Learning Nation!,
- qualified and dedicated teachers,
- high expectations of teachers,
- close monitoring of pupils’ progress in mathematics by teachers,
- pupils with good attitudes towards mathematics,
- pupils with high expectations of themselves,
- supportive home environments,
- parents with high expectations of their children, and finally
- a society that values high achievement.

**Concluding remarks**

The year 2004 saw a change in the political leadership of Singapore. In his maiden speech, our Prime Minister, Lee Hsien Loong has emphasized the need to make significant changes to the present education system, so as to allow for more diversity and curiosity. The most recent initiative mentioned is “Teach less, learn more”. In response to this initiative a wave of changes are anticipated and we await eagerly for them to swing into the system.

**References**


Regular Lecture

Curriculum Development Institute of Singapore (CDIS). (1994b). *Space trek through symmetry.* Singapore: Edutech Media Pte Ltd. [Computer software]


Curriculum Development Institute of Singapore (CDIS). (1995b). *Through the ages with congruency and similarity.* Singapore: Edutech Media Pte Ltd. [Computer software]


