Q9. In Fig. 12-21, a vertical rod is hinged at its lower end and attached to a cable at its upper end. A horizontal force \( \vec{F}_a \) is to be applied to the rod as shown. If the point at which the force is applied is moved up the rod, does the tension in the cable increase, decrease, or remain the same?

---

Increase. Consider torques about the reference point at the hinge. Only \( \vec{F}_a \) and \( T_{cable} \) exert non-zero torques. Since the moment arm for \( \vec{F}_a \) increases, its torque increases. This can be balanced only by an increase in \( T_{cable} \).

P17. In Fig. 12-32, a uniform beam of weight 500 N and length 3.0 m is suspended horizontally. On the left it is hinged to a wall; on the right it is supported by a cable bolted to the wall at distance \( D \) above the beam. The least tension that will snap the cable is 1200 N. (a) What value of \( D \) corresponds to that tension? (b) To prevent the cable from snapping, should \( D \) be increased or decreased from that value?

---

Recognize a statics problem with torques on one object: the beam. XFBFD for beam: \( R_x \) at left end, rightward \((R_x)\) is the x-component of the “reaction force” at the hinge); \( R_y \) at left end, upward; \( W \) at center of beam, downward (weight of beam; only one weight in this problem); \( T \) angled up/left at right end at angle \( \theta \) with horizontal \((T = 1200 \text{ N is known, with } \theta \text{ unknown})\).

Frequently it pays to tackle the torque equation first. Set reference point at left end, so that only \( W \) and \( T \) exert non-zero torques.

Torque eq: \( T \sin \theta L - W L/2 = 0 \). Solve for \( \sin \theta \) since \( T \) and \( W \) are known:

\[
\sin \theta = W/2T = (500 \text{ N})/(2 \cdot 1200 \text{ N}) = 0.2083 \Rightarrow \theta = 12.02^\circ.
\]

(a) We are asked for \( D \), but \( D = L \tan \theta = (3.0 \text{ m})(0.2129) = 0.639 \text{ m} \).

(b) Increased. From the torque equation, it’s \( T \sin \theta \) that must balance the fixed torque from \( W \). \( T \) decreases if \( \theta \) increases.

P25. In Fig. 12-40, what magnitude of (constant) force applied horizontally at the axle of the wheel is necessary to raise the wheel over an obstacle of height \( h = 3.00 \text{ cm} \)? The wheel’s radius is \( r = 6.00 \text{ cm} \), and its mass is \( m = 0.800 \text{ kg} \).

---

A statics problem, even though the wheel moves. We calculate the maximum force that doesn’t move the wheel, and an infinitesimal amount more force does move it.

XFBFD for wheel: A circle. \( mg \) down, at center; \( n_G \) upward, at bottom; \( n_E \) angled up/left, at contact with the edge of the step; \( \vec{F}_0 \) to right, at center. \( \vec{a} = 0 \). Conventional \( x\)-\( y \). We needed to distinguish normal-from-ground \( n_G \) from normal-from-edge \( n_E \).

If \( F_0 \) is gradually increased until the wheel moves, it starts to move as \( n_G \) goes to zero. Therefore we don’t include it in the equations, which apply when it is on the verge of moving.

We clearly will need the torque equation. After digesting it, we will see whether the \( x \) and \( y \) equations are even needed. So, start with torque equation. For the ref point, select the point where the least-known force is applied, the contact point with the edge. Since \( n_G = 0 \) and \( \tau_{\text{from } n_E} = 0 \), the only forces which exert torques are \( F_0 \) and \( mg \). The moment arm for \( F_0 \) is \((r - h) \). The moment arm for \( mg \) is the horizontal distance between the edge and the contact point with the horizontal surface. Call this \( \ell \).

Hmm. Consider the right triangle formed by the center of the wheel, the edge contact, and the point directly above the edge contact at a distance \( r \) above the bottom surface. Draw this, apply Pythagoras, and simplify to get \( \ell = \sqrt{h(2r - h)} \). So we can finally write the torque equation:

\[
mgl - F_0(r - h) = 0.
\]

Plugging numbers, we see that

\[
\ell = \sqrt{(3 \text{ cm})(2 \cdot 6 \text{ cm} - 3 \text{ cm})} = 5.196 \text{ cm} \quad \text{and} \quad r - h = 3 \text{ cm}.
\]

Hence

\[
F_0 = mg\ell/(r - h) = mg = (0.8 \text{ kg})(9.8 \text{ m/s}^2)(5.196 \text{ cm})/(3 \text{ cm}) = 13.58 \text{ N}
\]
**P37.** In Fig. 12-49, a uniform plank, with a length $L = 6.10 \text{ m}$ and a weight of $445 \text{ N}$, rests on the ground and against a frictionless roller at the top of a wall of height $h = 3.05 \text{ m}$. The plank remains in equilibrium for any value of $\theta \geq 70^\circ$ but slips if $\theta < 70^\circ$. Find the coefficient of static friction between the plank and the ground.

Statics problem with torque. Skelton says it should be a three-dot problem.

XFBD for plank: Line at $\theta = 70^\circ$. $mg$ downward, at center; $n_R$ at $\theta$ from the vertical (making it perpendicular to the plank), at the roller; $n_G$ upward, at base; $f_s$ to right, at base.

We note that $n_R$ has horizontal and vertical components, so that we will need it to get $n_G$ and $f_s$.

Start with torque eq. Select ground contact as ref point. Torques are due to $n_R$ and $mg$. The $n_R$ is applied perpendicular to the plank, so its moment arm is $\ell$, the distance up the plank to the roller – and I call it $\ell$ because a little trig is required. $h = \ell \sin \theta \implies \ell = h/\sin \theta$. The moment arm for $mg$ is the horizontal distance from the ladder-ground contact to the point on the ground directly below the com of the ladder. This moment arm is $(L/2) \cos \theta$. The torque eq is:

$$n_R(h/\sin \theta) - (mg)(L \cos \theta/2) = 0.$$  
Be sure you understand this.

$$n_R = mg(L/2h) \sin \theta \cos \theta = (445 \text{ N})(1)(0.9397)(0.3420) = 143.0 \text{ N}.$$  

We do need the $x$ and $y$ equations.

$x$-eq: $f_{s,max} - n_R \sin \theta = 0 \implies f_{s,max} = n_R \sin \theta = (143 \text{ N})(0.9397) = 134.4 \text{ N}.$

$y$-eq: $n_R \cos \theta = n_G - mg = 0 \implies n_G = mg - n_E \cos \theta = 445 \text{ N} - (143 \text{ N})(0.3420) = 396.1 \text{ N}.$

Since we are at the limit of static friction,

$$\mu_s = f_{s,max}/n_G = 134.4/396.1 = 0.339.$$  

**P45.** In Fig. 12-54, a lead brick rests horizontally on cylinders $A$ and $B$. The areas of the top faces of the cylinders are related by $A_A = 2A_B$; the Young’s moduli of the cylinders are related by $E_A = 2E_B$.

The cylinders had identical lengths before the brick was placed on them. What fraction of the brick’s mass is supported (a) by cylinder $A$ and (b) by cylinder $B$? The horizontal distances between the center of mass of the brick and the centerlines of the cylinders are $d_A$ for cylinder $A$ and $d_B$ for cylinder $B$.

(c) What is the ratio $d_A/d_B$?

(a) Recognize an elasticity problem. Since the brick is horizontal and the cylinders were originally the same length, both are compressed the same amount, so have the same strain $\Delta L/L$.

Per eq 12-23: $\Delta L/L = F/AE$, so

$$F_A/(E_AA_A) = F_B/(E_BA_B).$$  
Apply the values in the problem:

$$F_A/(2E_B \cdot 2 A_B) = F_B/(E_BA_B) \implies F_A = 4F_B.$$  

(a) $0.8$ (4/5) of the mass

(b) $0.2$ (1/5) of the mass

(c) An exercise in statics. Taking torques about the com, we see that

$$F_A d_A = F_B d_B \implies d_A/d_B = F_B/F_A = 1/4$$

**P39.** (Optional). For the stepladder shown in Fig. 12-51, sides $AC$ and $CE$ are each 2.44 m long and hinged at $C$. Bar $BD$ is a tie-rod 0.762 m long, halfway up. A man weighing 854 N climbs 1.80 m along the ladder. Assuming that the floor is frictionless and neglecting the mass of the ladder, find (a) the tension in the tie-rod and the magnitudes of the forces on the ladder from the floor at (b) $A$ and (c) $E$. (Hint: Isolate parts of the ladder in applying the equilibrium conditions.)

This is for civil engineer types (and physics majors, of course).

Recognize a statics problem, and take the hint of analyzing the two sections separately.

XFBD for section $AC$: Line sloped up/right. $n_A$ upward, at bottom; $T$ to right, halfway up; $mg$ down, at distance $d$ up the line (make it $d$ to keep it general, even though the distance is stated); $R_x$ to left, at top of ladder (use $R_x$ and $R_y$ as reaction force components, and $R_x$ must be leftward to balance the torque about the bottom); $R_y$ upward, at top (it must be upward: consider replacing the right section
of the ladder with a frictionless wall, and the left section would slide downward). \( \vec{a} = 0 \). Conventional xy.

XFBF for section EC: Line sloped up/left. \( n_E \) upward, at bottom; \( T \) to left, halfway up (reaction pair to the \( T \) on the right segment); \( R_x \) to right, at top of ladder (this is the reaction pair to the \( R_x \) on the right segment, so it has the same magnitude \( R_x \) and points in opposite direction); \( R_y \) downward, at top (again, Newton’s 3rd Law). \( \vec{a} = 0 \). Conventional xy.

We will get \( x \)-, \( y \)-, and torque equations for each section: 6 equations in 5 unknowns: \( n_A \), \( n_E \), \( T \), \( R_x \) and \( R_y \). Sounds fishy, but proceed and see if that clarifies. On the torque equations, setting the ref point at the top of the ladder will eliminate torque contributions from \( R_x \) and \( R_y \).

Also, define \( \theta \) at the angle between either section and the horizontal. Consider the triangle \( ACQ \), where \( Q \) is on the floor halfway between \( A \) and \( E \). The length of \( AQ \) is equal to that of \( BD \), 0.762 m, from similar triangles.

\[
\cos \theta = \frac{0.762 \text{ m}}{2.44 \text{ m}} = 0.3123 \Rightarrow \theta = 71.80^\circ.
\]

The six equations are:

\[
\begin{align*}
\text{x-eq for } AC: & \quad T - R_x = 0 \quad \text{Eq 1} \\
\text{y-eq for } AC: & \quad n_A + R_y - mg = 0 \quad \text{Eq 2} \\
\text{Torque eq for } AC: & \quad mg(L - d) \cos \theta + T(L/2) \sin \theta - n_A L \cos \theta = 0 \quad \text{Eq 3} \\
\text{x-eq for } EC: & \quad \text{redundant} \\
\text{y-eq for } EC: & \quad n_E - R_y = 0 \quad \text{Eq 4} \\
\text{Torque eq for } EC: & \quad -T(L/2) \sin \theta + n_E L \cos \theta = 0 \quad \text{Eq 5}
\end{align*}
\]

From Eq 1, \( R_x = T \), for whenever we get \( T \).

From Eq 4, \( R_y = n_E \).

Substitute these into eqs 2 and 3:

\[
\begin{align*}
\text{Eq 2'}: & \quad n_A + n_E = mg \\
\text{Eq 3'}: & \quad [2n_E \cot \theta](L/2) \sin \theta - n_A L \cos \theta = -mg((L - d) \cos \theta) \quad \text{The } [] \text{ is } T.
\end{align*}
\]

Clean up and rearrange Eq 3’:

\[
\begin{align*}
\text{Eq 3'':} & \quad n_A - n_E = mg(1 - d/L) \\
\text{Eqs 2'} \text{ and 3'' are easily solved.}
\end{align*}
\]

\[
\begin{align*}
\text{If } & \quad n_A = mg(1 - d/2L) = (854 \text{ N})(1 - (1.80/(2 \cdot 2.44))) = 539 \text{ N.} \\
\text{If } & \quad n_E = mg(d/2L) = (854 \text{ N})(1.80/(2 \cdot 2.44)) = 315 \text{ N.}
\end{align*}
\]

(a) \[ T = mgd \cot \theta /L = (854 \text{ N})(1.80 \text{ m})(0.3288)/(2.44 \text{ m}) = 207.1 \text{ N} \]

(b) \[ n_A = 539 \text{ N}, \text{ as found above.} \]

(c) \[ n_E = 315 \text{ N}, \text{ as found above.} \]

And: \( R_y = n_E = 359 \text{ N} \) and \( R_x = T = 207.1 \text{ N.} \)

My solution is correct; the answers in the back are for \( d/L = 0.75 \), which would correspond to climbing 1.80 m up a 2.40 m ladder segment.

Are you now ready to tackle that railroad bridge?