**P19.** Two particles are created in a high-energy accelerator and move off in opposite directions. The speed of one particle, as measured in the laboratory, is 0.650\(c\), and the speed of each particle relative to the other is 0.950\(c\). What is the speed of the second particle, as measured in the laboratory?

Let the first particle move at \(u = 0.650\, c\) to the right. Then particle 2 moves to the left. Take the lab frame as unprimed, and the first particle as the moving (primed) frame. Then \(v_x\) is the unknown velocity of particle 2 in the lab frame. Then \(v_x^2 = -0.950\, c\) is the given velocity of particle 2. It’s negative because it clearly goes in the \(-x\) direction.

\[
v_x = (v_x' + u)/(1 + v_xu/c^2) = (c)(-.95 + .65)/(1 + (-.95) \times .65) = -0.784\, c.
\]

**P23.** An imperial spaceship, moving at high speed relative to the planet Arrakis, fires a rocket toward the planet with a speed of 0.920\(c\) relative to the spaceship. An observer on Arrakis measures that the rocket is approaching with a speed of 0.360\(c\). What is the speed of the spaceship relative to Arrakis? Is the spaceship moving toward or away from Arrakis?

Take Arrakis as the lab (unprimed) frame and the spaceship as the moving (primed) frame. It’s clear the spacecraft is moving away from Arrakis, or else Arrakis would not measure a lower speed. Sketch the spacecraft to the right of Arrakis, firing to its left. This way, \(u\) is the (positive) velocity of the spacecraft, \(v_x = -0.360\, c\) is the velocity of the rocket as observed by Arrakis, and \(v_x' = -0.920\, c\) is the velocity of the rocket as observed by the spaceship.

\[
v_x = (u + v_x')/(1 + uv_x'/c^2).
\]

Solve for \(u\) and plug:

\[
u = (v_x - v_x')/(1 - v_xv_x'/c^2) = (c)(-.36 - (-.92))/(1 - (.36)(-.92)) = 0.837\, c.
\]

**P24.** Electromagnetic radiation from a star is observed with an earth-based telescope. The star is moving away from the earth with a speed of 0.600\(c\). If the radiation has a frequency of \(8.64 \times 10^{14}\) Hz in the rest frame of the star, what is the frequency measured by an observer on earth?

Doppler shift formula for moving away (Eq. 37.26):

\[
f = f_0\sqrt{(c - u)/(c + u)} = (8.64 \times 10^{14}\text{ Hz})\sqrt{0.4/1.6} = 4.32 \times 10^{14}\text{ Hz}.
\]

**P27.** A proton has momentum of magnitude \(p_0\) when its speed is 0.400\(c\). In terms of \(p_0\), what is the magnitude of the proton’s momentum when its speed is doubled to 0.800\(c\)?

Get the \(\gamma\) values for 0.4\(c\) and 0.8\(c\): 1.091 and 1.667. Since \(p = m\gamma v\), \(p_0 = m(1.091)(0.4)\) and \(p_1 = m(1.667)(0.8)\).

\[
p_1/p_0 = (1.667)(0.8)/[(1.091) \times (0.4)] = 3.06, \text{ so } p_1 = 3.06\, p_0.
\]

**P47.** The sun produces energy by nuclear fusion reactions, in which matter is converted into energy. By measuring the amount of energy we receive from the sun, we know that it is producing energy at a rate of \(3.8 \times 10^{26}\) W. (a) How many kilograms of matter does the sun lose each second? Approximately how many tons of matter is this (1 ton = 2000 lbs)? (b) At this rate, how long would it take the sun to use up all its mass?
Comment: the Sun could, in principle, only convert about 0.6% of its mass into energy, even if it converted all its hydrogen.

(a) \( e = mc^2 \) \Rightarrow m = e/c^2 = (3.8 \times 10^{28} \text{ J})/(3 \times 10^8 \text{ m/s})^2 = 4.22 \times 10^9 \text{ kg} \) each second. This is 4.2 million metric tons (a metric ton is 1000 kg), or about 4.64 million English tons (as measured in Earth’s gravitational field.)

(b) Pretending that the Sun could convert its entire mass,
\[
t = M_\odot/(4.22 \times 10^9 \text{ kg/s}) = (2 \times 10^{30} \text{ kg})/(4.22 \times 10^9 \text{ kg/s}) = 4.7 \times 10^{20} \text{ s} = 1.5 \times 10^{13} \text{ years}.
\]
In fact the Sun’s total lifetime is about 12 billion years, 1/1000 the calculated value; its age (4.6 billion years) is just under half its lifetime.

P52. A space probe is sent to the vicinity of the star Capella, which is 42.2 light-years from the earth. (A light-year is the distance light travels in a year.) The probe travels with a speed of 0.9930c. An astronaut recruit on board is 19 years old when the probe leaves the earth. What is her biological age when the probe reaches Capella?

Time dilation. \( \gamma = 8.467 \). Earth time is 42.2 ly/0.993c = 42.5 yr. Spaceship time \( t = t_0/\gamma = 42.5/8.467 = 5.0 \text{ yr} \). So: **24 years old**.

It would be longer because it would take 2.8 years at 1.0g to speed up and the same time to slow (if they actually want to stop at Capella), during which time \( \gamma \) would be less.

P55. The Large Hadron Collider (LHC). Physicists and engineers from around the world have come together to build the largest accelerator in the world, the Large Hadron Collider (LHC) at the CERN Laboratory in Geneva, Switzerland. The machine will accelerate protons to kinetic energies of 7 TeV in an underground ring 27 km in circumference. (For the latest news and more information on the LHC, visit www.cern.ch.) (a) What speed will protons reach in the LHC? (Since is very close to \( c \), write \( v = (1 - \epsilon)c \) and give your answer in terms of \( \epsilon \) [Book used \( \Delta \) where I use \( \epsilon \)] (b) Find the relativistic mass, of the accelerated protons in terms of their rest mass.

To work is SI, we need that 1 TeV = 10^{12} \text{ eV} = 10^{12} \cdot 1.602 \times 10^{-19} \text{ J} = 1.602 \times 10^{-7} \text{ J}$. Hence 7 TeV = 1.121 \times 10^{-6} \text{ J}.

The rest energy of the proton is
\[
m_p c^2 = (1.67 \times 10^{-27} \text{ kg})(3 \times 10^8 \text{ m/s})^2 = 1.50 \times 10^{-10} \text{ J}.
\]
When physicists speak of a 7 TeV proton, they mean its kinetic energy is 7 TeV, so that
\[
(7 \text{ TeV})/mc^2 = (\gamma - 1).
\]
\[
(7 \text{ TeV})/mc^2 = 1.121 \times 10^{-6}/1.50 \times 10^{-10} = 7476 = \gamma - 1.
\]
So, \( \gamma = 7477 \).

This addition of unity isn’t a big deal in this problem, but it would be for modest values of \( \gamma \).

From the definition of \( \gamma \), solve for \( v/c \) to get:
\[
v/c = \sqrt{\gamma^2 - 1}/\gamma = 0.99999999910 \text{ 5634}.
\]
Or:
\[
\epsilon = 8.94 \times 10^{-9}.
\]
(b) \( m_{rel} = m\gamma = 7477m \), where \( m \) is the rest mass.

By the way: A better way computationally to get \( \epsilon \) for large \( \gamma \) is by the Taylor series. You should easily get \( v/c = \sqrt{1 - 1/\gamma^2} \), so that:
\[
\epsilon = 1 - v/c = 1 - \sqrt{1 - 1/\gamma^2} = 1 - (1 - 1/2\gamma^2 - 1/8\gamma^4 - 1/16\gamma^6 + ...)
\]
\[
= 1/2\gamma^2 + 1/8\gamma^4 + 1/16\gamma^6 + 5/128\gamma^8 + ...
\]

P72. The French physicist Armand Fizeau was the first to measure the speed of light accurately. He also found experimentally that the speed, relative to the lab frame, of light traveling in a tank of water that is itself moving at a speed \( V \) relative to the lab frame is
\[ v = \frac{c}{n} + kV, \]

where \( n = 1.333 \) is the index of refraction of water. Fizeau called \( k \) the dragging coefficient and obtained an experimental value of \( k = 0.44 \). What value of do you calculate from relativistic transformations?

Comment: The quantity \( kV \) seems negligible compared to \( c/n \), but Fizeau was using interferometry to get the difference between \( v \) and \( c/n \), directly, which is \( kV \).

Let the flow velocity \( V \) be aiding the light. According to relativity

\[
    v = \frac{c}{n} + \frac{V}{1 + V/n^2}.
\]

Then \( v - c/n \) is

\[
    v - \frac{c}{n} = \frac{c}{n} + \frac{V}{1 + V/n^2} - \frac{c}{n} = \frac{V(1 - 1/n^2)}{1 + V/n^2}.
\]

The denominator is essentially unity, so Fizeau’s \( k \) should be

\[
    k = (1 - 1/1.333^2) = 0.437. \]

Fizeau was able to do that well without Starbucks or an iPhone.