Q13. What factors determine whether a material is a conductor of electricity or an insulator? Explain.

A conductor has the conduction band partly filled. This means the electrons can easily be excited to a higher state and can move freely under the influence of an electric field. An insulator has its highest occupied band completely filled; a completely filled band is more or less by definition, a valance band. The next band up is called the conduction band, but it’s empty. And it takes a lot of excitation (e.g., breakdown voltage) to kick an electron up there.

Q14. Ionic crystals are often transparent, whereas metallic crystals are always opaque. Why?

If it conducts electricity, it will respond to the \( \vec{E} \) field vector of an electromagnetic wave. This will either absorb or reflect the wave, so the material will be opaque. In ionic crystals, the captured electrons are usually held tightly by the negative ion. This means they are not free to move, so the material is not a conductor. This at least opens the door to the possibility of transparency.

Q15. Speeds of molecules in a gas vary with temperature, whereas speeds of electrons in the conduction band of a metal are nearly independent of temperature. Why are these behaviors so different?

It’s a matter of energy. The electrons in a metal occupy nearly all the states below the Fermi energy \( E_F \) (at low temperature, such as room temperature in this context) are occupied, and only a few electrons occupy higher states. These higher states are only slightly higher than \( E_F \). By contrast, gas molecules are essentially independent, so their energy is directly proportional to temperature.

P17. Density of NaCl. The spacing of adjacent atoms in a crystal of sodium chloride is 0.282 nm. The mass of a sodium atom is \( 3.82 \times 10^{-26} \) kg and the mass of a chlorine atom is \( 5.89 \times 10^{-26} \) kg. Calculate the density of sodium chloride.

Considering Fig 42.13 on pg. 1413, we can count how many \( \text{Na}^+ \) ions and how many \( \text{Cl}^- \) ions belong to this unit cell. We will then calculate its mass and divide by its volume.

Counting \( \text{Cl}^- \), ions there are:
- 8 corners, at 1/8 membership, plus:
- 6 faces, at 1/2 membership, for a total of
  - \( 1 + 3 = 4 \) \( \text{Cl}^- \) ions.

Counting \( \text{Na}^+ \), ions there are:
- 12 edges, at 1/4 membership, plus:
- 1 internal, at full membership, for a total of
  - \( 3 + 1 = 4 \) \( \text{Na}^+ \) ions.

Of course, they had to be equal.

So the mass of the unit cell is
\[
m = 4(3.82 + 5.89) \times 10^{-26} \text{ kg} = 3.884 \times 10^{-25} \text{ kg}.
\]

The volume is a cube whose side is \( 2 \cdot 0.282 \) nm = \( 5.64 \times 10^{-10} \) m.
\[
V = (5.64 \times 10^{-10} \text{ m})^3 = 1.79 \times 10^{-28} \text{ m}^3.
\]
\[ \rho = \frac{m}{V} = \frac{(3.884 \times 10^{-25} \text{ kg})}{(1.79 \times 10^{-28} \text{ m}^3)} = 2170 \text{ kg/m}^3. \]

For this structure, we could have used one of the eight smaller cubes, a slight simplification. Other crystal structures would require analyzing the full unit cell.

**P21.** The gap between valence and conduction bands in silicon is 1.12 eV. A nickel nucleus in an excited state emits a gamma-ray photon with wavelength \(9.31 \times 10^{-4} \text{ nm}\). How many electrons can be excited from the top of the valence band to the bottom of the conduction band by the absorption of this gamma ray?

We will need the photon (gamma-ray) energy, then see how many electrons it can promote.

\[ E_{\text{ph}} = \frac{(1240 \text{ eV} \cdot \text{nm})}{(9.31 \times 10^{-4} \text{ nm})} = 1.332 \text{ MeV}. \]

\[ N = \frac{(1.332 \text{ MeV})}{(1.12 \text{ eV})} = 1.19 \times 10^6 \text{ electrons}. \]

This is a common type of gamma-ray detector. By collecting the promoted electrons and measuring the charge, the amount of energy deposited can be determined—frequently to around 0.1% accuracy.

**P30.** (a) Suppose a piece of very pure germanium is to be used as a light detector by observing, through the absorption of photons, the increase in conductivity resulting from generation of electron-hole pairs. If each pair requires 0.67 eV of energy, what is the maximum wavelength that can be detected? In what portion of the spectrum does it lie? (b) What are the answers to part (a) if the material is silicon, with an energy requirement of 1.14 eV per pair, corresponding to the gap between valence and conduction bands in that element?

The key concept is that it takes a photon with sufficient energy (a sufficiently short wavelength) to create a pair.

(a) Since \( E_{\text{ph}} = 0.67 \text{ eV} \), \( \lambda = \frac{1240}{0.67} = 1851 \text{ nm} \), in the IR.

(b) Since \( E_{\text{ph}} = 1.14 \text{ eV} \), \( \lambda = \frac{1240}{1.14} = 1088 \text{ nm} \), still in the IR.

**P45.** Spectral Lines from Isotopes. The equilibrium separation for NaCl is 0.2361 nm [for an isolated molecule]. The mass of a sodium atom is \(3.8176 \times 10^{-26} \text{ kg}\). Chlorine has two stable isotopes, \(^{35}\text{Cl}\) and \(^{37}\text{Cl}\) that have different masses but identical chemical properties. The atomic mass of \(^{35}\text{Cl}\) is \(8.8068 \times 10^{-26} \text{ kg}\) and the atomic mass of \(^{37}\text{Cl}\) is \(6.1384 \times 10^{-26} \text{ kg}\). (a) Calculate the wavelength of the photon emitted in the \(\ell = 2 \rightarrow \ell = 1\) and \(\ell = 1 \rightarrow \ell = 0\) transitions for Na\(^{35}\text{Cl}\). (b) Repeat part (a) for Na\(^{37}\text{Cl}\). What are the differences in the wavelengths for the two isotopes?

We need Eq 42.3 \( E_l = l(l+1)\hbar^2/2I \) and Eq 42.6 \( I = m_r r_0^2 \), along with the definition of the reduced mass.

(a) Plugging numbers for \(^{35}\text{Cl}\):

\[ m_r = \frac{(3.8176)(5.8068)/(3.8176 + 5.8068)}{10^{-26} \text{ kg}} = 2.33033 \times 10^{-26} \text{ kg}. \]

\[ I = \frac{(2.33033 \times 10^{-26} \text{ kg})(2.361 \times 10^{-10} \text{ m})^2}{10^{-46} \text{ kg m}^2}. \]

\[ \hbar^2/2I = \frac{(1.05457 \times 10^{-34})^2}{(2 \times 1.28394 \times 10^{-46} \text{ kg m}^2)} = 4.33087 \times 10^{-24} \text{ J} = 2.70313 \times 10^{-5} \text{ eV}. \]

For \(\ell = 2 \rightarrow \ell = 1\), the energy change is \(2 \cdot 3 - 1 \cdot 2 = 4\) times this basic unit, so \(E_{\text{ph}} = 4 \cdot (2.70313 \times 10^{-5} \text{ eV}) = 1.08125 \times 10^{-4} \text{ eV}. \)

\[ \lambda = \frac{1240}{1.08125 \times 10^{-4}} = 11.4682 \text{ mm}. \]

For \(\ell = 1 \rightarrow \ell = 0\), the energy change is \(1 \cdot 2 - 0 \cdot 1 = 2\) times this basic unit, so wavelength is doubled.

\[ \lambda = 22.9364 \text{ mm}. \]

(b) Plugging numbers for \(^{37}\text{Cl}\):
\( m_r = 2.35375 \times 10^{-26} \text{ kg.} \)
\( I = 1.31206 \times 10^{-46} \text{ kg m}^2. \)
\( \frac{\hbar^2}{2I} = 2.64519 \times 10^{-5} \text{ eV.} \)

For \( \ell = 2 \rightarrow \ell = 1, \) \( E_{ph} = 4 \cdot (2.64519 \times 10^{-5} \text{ eV} = 1.05808 \times 10^{-4} \text{ eV.} \)

\( \lambda = 1240/1.08125 \times 10^{-4} = \textbf{11.7194 mm} \)

For \( \ell = 1 \rightarrow \ell = 0, \)
\( \lambda = \textbf{23.4388 mm}. \)

The difference for the 11-mm line is \( \Delta \lambda = 0.2512 \text{ mm}, \) and twice that for the 23-mm line. Perhaps more relevant is the percentage change of 2.17%, equal for both. The point is that the isotopic separation is detectable with modest resolution in molecular spectra; in atomic spectra, it required high resolution.