The Use of Optical Pumping in Determining the Nuclear Spins of Rb\textsuperscript{85} and Rb\textsuperscript{87}

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Using TeachSpin’s OP1-A optical pumping RF spectrometer, we probed the low and high field Zeeman effects of two rubidium isotopes, Rb\textsuperscript{85} and Rb\textsuperscript{87}, with the goal of determining the nuclear spins, \( I \), of each. To this end, we first determined \( g_F \) of each isotope through low field optical pumping RF resonance spectroscopy. Next, we determined \( I \) through a simple relationship between \( g_F \) and \( I \). Finally, we examined the high field resonance spectra to assign the nuclear spins to each isotope. In Rb\textsuperscript{85}, \( g_F \) was determined to be \( 0.38 \pm 0.07 \), yielding an \( I \) of \( 3.07 \pm 0.07 \). In Rb\textsuperscript{87}, \( g_F \) was determined to be \( 0.42 \pm 0.07 \), yielding an \( I \) of \( 1.89 \pm 0.07 \). Thus, we found \( g_F \) and \( I \) to be different within error and similar to known and published nuclear spin values for both isotopes. The main source of uncertainty in this experiment has been in the difficulty of obtaining the calibration of the interface between the data acquisition program reported volts and the actual volts applied through the Helmholtz coils; the speed of the voltage sweep yielded uncertainty while reading voltage output from the multimeter.

I. INTRODUCTION

Since the development of modern atomic theory by John Dalton in the early 1800’s, scientists have been fascinated with the atom. To insure a thorough understanding of the intricacies of the atom, the field of atomic physics was born. However, this precise study of atoms has been no easy task. To fully probe the properties of this basic unit of matter is made difficult by the fact that it is so small and difficult to observe. Through recent technical developments and the natural curiosity and creativity of physicists, the study of atomic physics has been made easier over time. Of interest is Alfred Kastler’s Nobel Prize winning discovery of the phenomenon of optical pumping in 1950, which, through a combination of clever techniques, yields a window into the subatomic world.

Because determining the nuclear spin of atoms is crucial in developing a thorough understanding of atoms, we conduct an experiment in which we will determine the nuclear spins of Rb\textsuperscript{85} and Rb\textsuperscript{87} by combining optical pumping with resonance radiation. Rubidium was chosen for study because of its hydrogen-like properties; it can be considered a one-electron atom because the rest of its electron shells are fully closed and considered to be core electrons. The result of this experiment is greatly important; this experiment shows that it is possible to determine the nuclear spin of atoms, which is valuable not only in backing up atomic theory, but also in the development of future experiments to further the understanding of atoms. The success of this experiment in determining nuclear spin will allow physicists to add this to their toolbox for studying atoms and is a valuable contribution not only to the field of atomic physics, but also quantum physics and physical chemistry. Furthermore, this experiment also helps in the solidification of the understanding of optical pumping which is the basis of all lasers and, as such, is a huge contribution to technological development.

The structure of alkali atoms will first be discussed in Sec. II A, which will form a background understand-}

ing for the discussion of how this structure changes in the presence of a magnetic field in Sec. II B. This theory section will also include an introduction to optical pumping in Sec. II C followed by an introduction to RF resonance spectroscopy in Sec. II D. After discussing the theory, the experimental design of the apparatus used, the method of determining both \( g_F \) and \( I \), and the method of assigning nuclear spins to each isotope will be introduced in Sec. III A, Sec. III B, and Sec. III C, respectively. The results of the experiment will then be presented in Sec. IV, followed by a discussion of the results in Sec. V.

II. THEORY

A. Structure of Alkali Atoms

Alkali atoms can all be approximated to have the same atomic structure as that of a hydrogen atom due to the fact that the electrons in all of the inner shells are paired and thus can be completely ignored within the scope of this experiment. Like hydrogen’s atomic structure, alkali atoms all have only one single outer electron. Rubidium, with electronic configuration of

\[
1s^22s^22p^63s^23p^63d^{10}4s^24p^65s^1
\]

is no different and has its properties characterized by the single electron in the 5s shell.

This single outer shell electron can be characterized by its orbital angular momentum \( L \) and its spin angular momentum \( S \), which can be combined to form the total non-nuclear angular momentum \( J \), with

\[
J = L + S. \tag{1}
\]

By vector addition, the allowed \( J \) values are \( J = S, L + S - 1, L + S - 2, ..., |L - S| \). In the ground state, the valence electron is in an \( s \) orbital, so \( L = 0 \) and since
each electron has $S = \frac{1}{2}$ and since there is only one valence electron, rubidium has $S = \frac{3}{2}$. Thus $J = \frac{1}{2}$ and the spectroscopic term is $2S_{1/2}$ for the ground state rubidium atom, where spectroscopic notation is written in the form of $2S + 1L_J$. However, in rubidium’s first excited state, the valence electron is in a $d$ orbital, so $L = 1$ while it remains that $S = \frac{3}{2}$. Thus in the first excited state $J = \frac{3}{2}$ and $J = \frac{1}{2}$, yielding two $P$ states with different energies, namely $2P_{3/2}$ and $2P_{1/2}$. The different energies occupied by these spectroscopic terms is known as the fine structure of the atom. It is worth noting that the energy difference between the ground state and either first excited state is much greater than the energy difference between the two first excited states.

Like electrons, nuclei also have spin angular momentum $I$ that vary with different nuclei. The total angular momentum of the atom can be summed up as $F$, with $F = J + I$. 

By vector addition, the allowed $F$ values are $J + I, J + I - 1, J + I - 2, ..., |J - I|$. This allows for further splitting of energies within spectroscopic terms, yielding what is known as the hyperfine structure. If one assumes that for a given isotope of rubidium, $I = 1/2$, then both $2S_{1/2}$ and $2P_{1/2}$ will each have one hyperfine splitting, resulting in two energy levels each, corresponding to $F = 1$ and $F = 0$ while $2P_{3/2}$ will also have one hyperfine splitting, resulting in two energy levels, corresponding to $F = 2$ and $F = 1$.

### B. Effect of Magnetic Field on Alkali Atomic Structure

Each of the angular momentum vectors are associated with a magnetic dipole moment, with $\mu_L, \mu_S, \mu_J, \mu_I, \mu_F$ associated with $L, S, J, I, F$ respectively. $\mu_J$ is the result of the spin coupling of $\mu_L \cdot \mu_S$ while $\mu_F$ is the result of the spin coupling of $\mu_J \cdot \mu_I$. Because the angular momentum of the atom is associated with magnetic dipole moments, a weak external magnetic field changes the energy levels within the atom, producing further splitting of energy levels, known as the Zeeman effect. The regime of the Zeeman effect is when the resultant spin energy levels have an energy difference much smaller than the hyperfine splitting energy level differences. The result of this weak external magnetic field $B$ can be modeled by the Hamiltonian

$$\mathcal{H} = h\omega I \cdot J - \frac{\mu_J}{J} J \cdot B - \frac{\mu_I}{I} IB,$$

where $h$ is Planck’s constant and $a$ is an experimentally determined constant that varies with electronic configuration. This Hamiltonian yields splits of each $F$ level into $2F + 1$ sublevels with energy differences determined by the strength of the applied field.

The resultant interaction energy between the applied magnetic field and the atom can be expressed by the vector model of Eq. 3, above, and is found to be

$$W = g_F \mu_B BM,$$

where $\mu_B$ is the Bohr magneton, $B$ is the magnitude of the magnetic field, $M$ is the component of the electron spin along the magnetic field,

$$g_F = \frac{g_J}{2F(F+1)} \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)},$$

and

$$g_J = \begin{cases} 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} & \text{if } J = 1/2, \\ 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} & \text{if } J = 3/2. \end{cases}$$

It is worth noting that $g_J$ is known as the Lande $g$-factor and is equal to 2 in the case of rubidium when $J = S = 1/2$.

However, Eq. 4 holds only if $W$ is small and the energy levels depend linearly on the magnetic field. In high magnetic fields, the equation becomes quadratic and can be solved by diagonalizing Eq. 3, which results in the Breit-Rabi equation

$$W(F,M) = -\frac{\Delta W}{2(2I+1)} \frac{\mu J}{I} BM \pm \frac{\Delta W}{2} \left[ 1 + \frac{4M}{2I + 1} x^2 + x^2 \right]^{1/2},$$

where

$$x = (g_J - g_I) \frac{\mu_B}{I} B$$

and $g_I = -\frac{\mu_I}{I} \frac{1}{I} \frac{1}{\mu_B}$ and $W$ is the interaction energy and $\Delta W$ is the hyperfine energy splitting. When the Breit-Rabi equation is plotted with $W/\Delta W$ on the vertical axis and $x$ on the horizontal axis, the plot consists of three regions: the low field ($x$ very close to 0) Zeeman region, where the splitting is linear with $B$, the Paschen-Back region ($x > 2$), where the splitting is also linear with $B$, and the intermediate field region (between the Zeeman region and the Paschen-Back region), where the energy levels are no long linear with $B$. This experiment deals with the Zeeman region and the intermediate field region.

### C. Optical Pumping

For the splitting of the hyperfine structure by an applied external magnetic field, a set of selection rules enforce the absorption or emission of light due to electric dipole transitions. For this system, the selection rules are as follows: $\Delta S = 0, \Delta J = 0, \pm 1, \Delta L =$
0, ±1 but not \( L = 0 \) to \( L = 0, \Delta F = 0, \pm 1 \), and \( \Delta M = 0, \pm 1 \). However, due to conservation of angular momentum, the selection rule for absorption changes in regards to \( M \) in our system because the absorption would be that of circularly polarized light, causing the selection rule for absorption in regards to \( M \) to change to only having +1 or −1 allowed, but not both. Furthermore, magnetic dipole transitions occur with selection rules of \( \Delta F = 0, \pm 1 \) and \( \Delta M = 0, \pm 1 \) with the transition occurring dependent on the apparatus’s RF magnetic field orientation with respect to the applied external magnetic field. At a perpendicular orientation with frequencies under the gigahertz range, the magnetic dipole transition selection rules become \( \Delta F = 0 \) and \( \Delta M = \pm 1 \). Due to the conditions set up by this experiment, the selection rules are such that the energy levels occupied are limited to \( M = +2 \) and \( M = -2 \) since absorption is forbidden from these states. Under these conditions, after the incident light is applied for the right length of time, the system will be optically pumped and no transitions can occur. The system will now be at a non-thermodynamic equilibrium state.

D. RF Resonance Spectroscopy

From Eq. 4 it can be seen that the energy difference between adjacent levels is, in terms of frequency \( \nu \) in Hz,

\[
\nu = g_F \mu_B B / h \tag{9}
\]

because adjacent level \( M \) differ only by 1. Due to optical pumping, a majority of the population now exists in either the \( M = +2 \) or \( M = -2 \) state. Then, when the RF is turned on, the selection rules change again to allow for transitions back to thermal equilibrium to occur. Now that transitions back to thermal equilibrium is allowed, the incident light intensity will drop when a resonance is encountered at a specific magnetic field.

III. EXPERIMENTAL

A. Apparatus

Apparatus consists of a rubidium discharge lamp, an interference filter, a linear polarizer, a quarter wave plate, a rubidium gas cell, a focusing lens, and an optical detector for the optical pathway. The rubidium discharge lamp outputs two main wavelengths of light: 780 and 795 nm. However, an interference filter is placed right after the lamp to ensure that only 795 nm light is allowed through the optical path. A linear polarizer follows the filter to linearly polarize the light. A quarter wave plate follows to circularly polarize the light to right hand or left hand polarized. The light enters the rubidium gas cell and exits to a lens that focuses the light towards an optical detector. Three helmholtz coils are placed around the gas cell to allow magnetic field manipulation in two dimensions and magnetic field sweeping in of this two dimensions.

The apparatus is connected to a control panel that allows the control of an RF amplifier, the helmholtz coils, and the optical detector. An oscilloscope and an RF generator are also attached to the control panel to read data from the helmholtz coils and optical detector and generate an RF into the rubidium gas cell of a desired frequency.

B. Low Field RF Resonance Spectroscopy

A \( B_0 \) dip was first found, which occurs when the gas cell is immersed in a net magnetic field of 0. The RF frequencies used in this part of the experiment range from 50 \( kHz \) to 150 \( kHz \). At each frequency, the magnetic field is swept about the \( B_0 \) dip to elicit two other dips, one for Rb\(^{85} \) and one for Rb\(^{87} \). The \( B \) value at each dip for each frequency are recorded.

C. High Field RF Resonance Spectroscopy

The starting magnetic field is increased to about 17\( \mu T \) and the RF generator frequency is slowly modulated from 1\( MHz \) to 10\( MHz \) until dips are observed. The magnetic field value corresponding to each dip is recorded along with the associated applied RF frequency.

IV. RESULTS

\( g_F \) was found to be 0.42 and 0.38 for the inner peak and outer peak respectively by using the trendline equations in Fig. 1 of the form of Eq. 9. This yields \( I = 1.89 \) and \( I = 3.07 \) for the inner peak and outer peak respectively because reduction of Eqs. 1, 2, 5, and 6 yield \( g_F = \frac{1}{I} \) for \( L = 0 \) and \( S = 1/2 \). Fig. 2 shows how the peaks were labeled while which isotope was responsible for which peak was unknown. To determine which peak was which isotope, Fig. 3 was used. Matching this spectra with the lab manual as being \(^{85}\)Rb and finding another quadratic high field spectra at a higher RF but same magnetic field revealed that the inner peak is \(^{87}\)Rb and the outer peak is \(^{85}\)Rb. This conclusion was made because higher frequency means that \( g_F \) is also increased. Thus, \(^{85}\)Rb must be the peak with the smaller \( g_F \). Thus, \(^{85}\)Rb has a \( g_F \) of 0.38 and an \( I = 3.07 \) and \(^{87}\)Rb has a \( g_F \) of 0.42 and an \( I = 1.89 \).

V. DISCUSSION
FIG. 1. Plot of frequency versus magnetic field for the low field Zeeman effect for the rubidium gas cell. Trendlines were fit to determine the slope, which is used to calculate $g_F$.

FIG. 2. Plot of the low field Zeeman effect spectra of arbitrary absorbance versus arbitrary time for the rubidium gas cell at 60 kHz. The peaks are referred to as shown above.

FIG. 3. Plot of the high field quadratic spectra of arbitrary absorbance versus arbitrary time for the rubidium gas cell at 3.8874 MHz.