

-A brief introduction

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I on

A<sup>2</sup>coustic

Waves

that assumes good assimilation  
of

the notes on sound waves

# Are there electrostatic, longitudinal, dispersionless waves in a plasma? Yes, sort of!

Try this. Think of ion motion

$$NII \quad M n_i \left[ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \frac{d\mathbf{v}}{dx} \right] = e n_i [\mathbf{E} + \mathbf{v} \times \mathbf{B}]$$

$E_0 = 0$  consider  
only perturbed E

$B = 0$  unmagnetized  
⇒ Electrostatic Waves only!

Maxwell

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_0 = \frac{\rho}{\epsilon_0} (n_i - n_e)$$

(Continuity)

$$\frac{\partial n}{\partial t} + \frac{d}{dx} (n_i v) = 0$$

Maxwell

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_0 \quad \nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$
$$\frac{\partial \Phi_E}{\partial t} = \frac{1}{\epsilon_0} \frac{\partial \mathbf{E}}{\partial t}$$
$$\frac{\partial \Phi_B}{\partial t} = \frac{1}{\mu_0} \frac{\partial \mathbf{B}}{\partial t}$$

Your job: obtain  $n_1$  and  $\phi_1$  from continuity and Poisson to obtain the dispersion relation, an equation of the form,  $\mathcal{D}(\omega, k)\phi_1 = 0$

back to NE ch

$$v \rightarrow v,$$

$$n \rightarrow n_0 + n_1 \quad (\text{forget } i \dots \text{ extra sub. when needed})$$

$$E \rightarrow E_1 \quad \text{all in } \hat{x} \dots \text{ longitudinal}$$

$$\delta n_1 \rightarrow n_1(x, t) \rightarrow n_1 e^{i(kx - \omega t)} + \text{perturbed quantities}$$

$$\frac{\partial}{\partial t} n_1 \rightarrow -i\omega n_1, \quad \frac{d}{dx} n_1 \rightarrow ik n_1.$$

Linearization  $E = -\nabla \phi = -\frac{d}{dx} \phi, \quad E_1 = -ik \phi_1$

NE

$$M(n_1 + n_0) \left[ \frac{\partial}{\partial t} v_1 + v_1 \frac{d}{dx} v_1 \right] = e(n_0 + n_1) [E_1]$$

$$\underline{Mn_0 \frac{\partial v_1}{\partial t} + Mn_1 \frac{\partial v_1}{\partial t} + Mv_1 \frac{d}{dx} v_1 + Mn_0 v_1 \frac{d}{dx} v_1} = en_0 E_1 + en_1 E_1$$

this equation is, to a 1st approx...

$$-Mn_0 \omega v_1 = en_0 E_1 = -en_0 ik \phi_1$$

$$\boxed{v_1 = \frac{en_0 \phi_1}{M\omega}}$$

Q

make sense??  
 $\omega v_1 = \frac{e k \phi_1}{M}$  units ok..

Not going to do all the algebra! But after  
eliminating  $n_1, \gamma_1, \phi_1$

$$k^2 \phi_1 = \left( \frac{e^2 n_0 \omega^2}{\epsilon_0 M} \right) \phi_1 - \left( \frac{e^2 n_0}{\epsilon_0 R T e} \right) \phi_1$$

remember FTO Eqs?

$$\left[ k^2 - \frac{\Omega_p^2 - k^2}{\omega^2} + \frac{1}{\lambda_0^2} \right] \phi_1 = 0$$

now that's a dispersion relation...

$$\omega^2 = \frac{\lambda_0^2 \Omega_p^2 - k^2}{k^2 \lambda_0^2 + 1}$$

$$k \ll \frac{1}{\lambda_0}, \omega \approx c_s k$$

OK, there it is - the dispersion relation, what we will test in the lab

