A brief introduction

2

# Ion Acoustic Waves

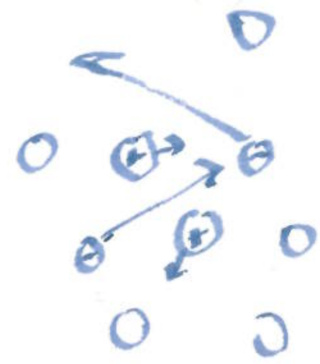
that assumes good assimilation

of

the notes on sound waves

Are there Electrostatic Longitudinal 'acoustic' waves in a plasma ?

A: Yes!




Try this. Think of ion motion

N II $\quad M n_i\left[\frac{\partial v}{\partial t} + v\frac{dv}{dx}\right] = e n_i\left[E + u_x \times B\right]$

Recap

300Hz 440H

$\lambda_{mfp} \ll \lambda_s$ for usual propagation

440H $\quad \lambda = \frac{v}{f} = \frac{330}{440} \sim .75m$

$\lambda_{mfp}$ @STP

$\frac{\lambda_s}{2} \sim 35cm$ bigger & again

$\lambda = \frac{1}{n\sigma}$

760 torr @STP

@STP... rule of thumb $N_2$

$\lambda \sim \frac{5cm}{P[torr]}$ i.e. a 1 torr 5cm

$\therefore \frac{\lambda_{mfp}}{\lambda_s} \sim \frac{5\times10^{-2}cm}{75cm} \ll 1$ well satisfied

$E_0 = 0$ consider only perturbed E

$B = 0$ unmagnetised

& Electrostatic Waves only!

we'll get

$v_1 = \frac{e k}{M\omega}\phi_1$

o o o Can you 'see it'?


understanding is light. knowing & understanding differ! aim at understanding and you'll get knowing in the bargain

make strong ... & BE READY TO REACH OUT TO some ...

Maxwell

$\nabla\cdot E = \rho/\epsilon_0 = \frac{e}{\epsilon_0}(n_i - n_e)$

what can we learn from

$\nabla^2\phi_1 = \frac{e}{\epsilon_0}n_1 - \frac{n_0 e^2}{\epsilon_0 kT_e}\phi_1$

Maxwell

$\nabla\cdot\vec{E} = \rho/\epsilon_0 \qquad \nabla\cdot\vec{B} = 0$

$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \nabla\times\vec{B} = \mu_0\vec{J} + \frac{1}{\mu_0\epsilon_0}\frac{\partial\vec{E}}{\partial t}$

$\dot{\Phi}_B \rightarrow E \qquad \dot{\Phi}_E \rightarrow B$

Continuity

$\frac{\partial n}{\partial t} + \frac{d}{dx}(n_i v) = 0$

" can you see it? "

$n_1 = n_0\frac{k v_1}{\omega}$

" what does it say"

$n_i, v, E$ but $n_e$!

↳ Assume in equil! $\quad n_e = Ce^{e\phi/kT_e}$

$\frac{dn_e}{d\phi} = \frac{Ce}{kT_e}e^{e\phi/kT_e}$

T.E. for Electrons


Q what is $n_e(x)$

$\frac{dn}{d\phi} = \frac{n_0 e}{kT_e}$

back to NII etc

$v \to v_1$
$n \to n_0 + n_1$ (for e & i ... extra sub. when needed)
$E \to E_1$ all in $\hat{x}$ ... longitudinal

$\forall\, n_1 \to n_1(x,t) \to n_1 e^{i(kx-\omega t)}$ $\forall$ perturbed quantities

$$\frac{\partial}{\partial t} n_1 \to -i\omega n_1 \quad \frac{d}{dx} n_1 \to ikn_1$$

Linearization $E = -\nabla\phi = -\frac{d}{dx}\phi \quad E_1 = -ik\phi_1$

NII

$$M(n_1+n_0)\left[\frac{\partial}{\partial t}v_1 + v_1\frac{d}{dx}v_1\right] = e(n_0+n_1)\left[E_1\right]$$

$$\underline{Mn_0\frac{\partial v_1}{\partial t}} + Mn_1\frac{\partial}{\partial t}v_1 + Mn_1v_1\frac{d}{dx}v_1 + Mn_0v_1\frac{d}{dx}v_1 =$$
$$en_0E_1 + en_1E_1$$

this equation is, to a 1st approx...

$$-M\cancel{n_0}i\omega v_1 = en_0E_1 = -e\cancel{n_0}ik\phi_1$$

$$\boxed{v_1 = \frac{ek\phi_1}{M\omega}}$$ ...... Q make sense?? $\omega v_1 = \frac{ek\phi}{M}$ units ok...

Continuity becomes

$$i\omega n_1 = n_0ikv_1$$

$$\boxed{n_1 = n_0\frac{kv_1}{\omega}}$$ ...... Q Can't you just "see" that $\frac{kv}{\omega}$ is dimensionless?

so, what about Poisson's Eq

$$\frac{d}{dx} \cdot E_1 = \frac{e}{\epsilon_0}(n_{i0} + n_{i1} - n_{e0} - n_{e1})$$

looks like we are in trouble!!! Just a first order quantity on the lhs with 0th & 1st quantities on rhs

$$ikE_1 = -i^2k^2\phi_1 = \frac{e}{\epsilon_0}((n_{i0} - n_{e0}) + (n_{i1} - n_{e1}))$$

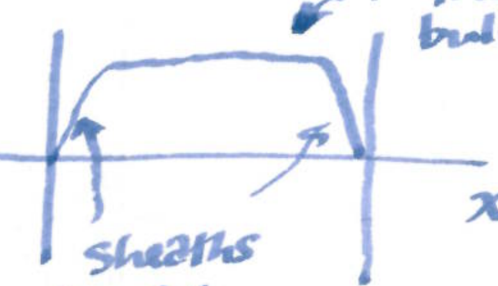
$\leftarrow \frac{n_{e0} e\phi_1}{kT_e}$ !

0 the plasma is neutral in this approximation

this is plasma behavior "quasi-neutral"

Why naming a good thing?

14921776 2011




sheaths are set up to maintain quasi-neutrality

very weird non-equilibrium non-linear behavior

$$\boxed{k^2\phi_1 = \frac{e}{\epsilon_0}n_1 - \frac{n_0 e^2}{\epsilon_0 kT_e}\phi_1} = \frac{e}{\epsilon_0}n_1 - \frac{\phi_1}{\lambda_D^2}$$

$$\boxed{v_1 = \frac{ek}{m\omega}\phi_1}$$

$$\boxed{n_1 = n_0\frac{kv_1}{\omega}}$$

$(\quad)\phi_1 = 0$

? "dispersion relation"

why is it called that?

<u>Not</u> going to do all the algebra! But after eliminating $n_1, v_1, \phi_1$

$$k^2\phi_1 = \frac{e^2 n_0}{\epsilon_0 M}\frac{k^2}{\omega^2}\phi_1 - \frac{e^2 n_0}{\epsilon_0 k T_e}\phi_1$$

? ?

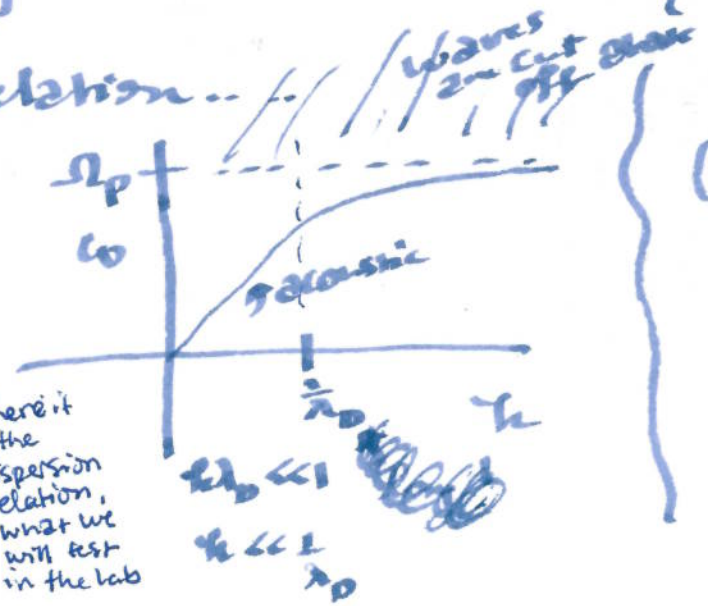
remember FTOEg's?

$$\left[k^2 - \frac{\Omega_p^2}{\omega^2}k^2 + \frac{1}{\lambda_D^2}\right]\phi_1 = 0$$

now that's a dispersion relation...

$$\omega^2 = \frac{\lambda_D^2\Omega_p^2 k^2}{k^2\lambda_D^2 + 1}$$

$k \ll \frac{1}{\lambda_D}$ $\omega \approx C_s k$




OK, there it is - the dispersion relation, what we will test in the lab

$$n_1 = n_0\frac{k}{\omega}\frac{ek}{M\omega}\phi_1$$

$$= \frac{en_0k^2}{M\omega^2}\phi_1$$

$$\frac{e}{\epsilon_0}n_1 = \frac{e^2n_0}{\epsilon_0 M\omega^2}k\phi_1$$

$$\boxed{\Omega_p^2 \equiv \frac{n_0e^2}{\epsilon_0 M}} \quad \omega_p^2 \equiv \frac{n_0e^2}{\epsilon_0 m}$$

$$\boxed{\lambda_D^2 = \frac{\epsilon_0 k T_e}{e^2 n_0}}$$

$$\lambda_D\Omega_p = C_s!$$

$$\boxed{C_s = \sqrt{\frac{kT_e}{M}}}$$

$$\left(1 - \frac{\Omega_p^2}{\omega^2}\right)k^2 = -\frac{1}{\lambda_D^2}$$

$$\frac{\Omega_p^2 k^2}{\omega^2} = k^2 + \frac{1}{\lambda_D^2}$$

$$\frac{\omega^2}{k^2\Omega_p^2} = \frac{1}{k^2 + \frac{1}{\lambda_D^2}}$$

$$\omega^2 = \frac{k^2\Omega_p^2}{k^2 + \frac{1}{\lambda_D^2}}$$