Resolving the hyperfine structure of rubidium
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In this experiment we aimed to find the hyperfine splitting in $^{85}$Rb and $^{87}$Rb in both the ground state and first excited state. For $^{85}$Rb and $^{87}$Rb in the ground state we found the frequency difference between the two hyperfine state to be $2850\text{MHz} \pm 219\text{MHz}$ and $6407\text{MHz} \pm 493\text{MHz}$ respectively, both of which agree with the literature. For the first excited state of $^{85}$Rb we found the difference in energy states to be $138\text{MHz} \pm 15\text{MHz}$ and $270\text{MHz} \pm 25\text{MHz}$ which also agrees with the literature. The resolving power for this experimental setup was not enough to resolve the hyperfine structure of $^{85}$Rb. We also confirmed the theory of Doppler Broadening by calculating the temperature necessary to produce a Full Width Half Max measurement consistent with the data collected.

I. INTRODUCTION

Laser Spectroscopy can be used to learn about the structure of atoms as well as their constituent electron energy levels. Spectroscopy was instrumental in the development of quantum mechanics as it provided a basis through which theories could be tested. The spectroscopy of different atoms helped develop more advanced theories in cosmology as well as nuclear physics as it provides an relatively simple way to probe the tiny. In this experiment we aimed to develop and confirm our understanding of the two Rubidium isotopes, $^{85}$Rb and $^{87}$Rb. Section II aims to develop and foundation of understanding for both the ground state and first excited state hyperfine splitting in both $^{85}$Rb and $^{87}$Rb. Section III discusses issues that arise because of Doppler Broadening as well as develop some theory to help better understand it. Section IV explains the experimental setup as well as the significance of each part in the experimental setup. Section V is broken up into three subsections that discuss results from the three main components of our experiment and Section VI concludes the paper with summation of the results.

II. UNDERSTANDING HIGH RESOLUTION LASER SPECTROSCOPY

A typical atom consists of a nucleus and electrons. These electrons can occupy different energy states that are filled based on the Pauli Exclusion principle. Electrons that are on the outer most "shell" can be temporarily excited to an energy state that is above their stable state by absorbing a photon with an energy that is equal to the difference between the two energy states. That is to say that

$$\Delta E = E_2 - E_1 = hf,$$  \hfill (1)

where $f$ is the frequency of light required to create a transition from $E_1 \to E_2$. This alone will give a basic understanding of FIG. 1, where the frequency of a photon required to excite an electron from $5S_{1/2} \to 5P_{3/2}$ is $384 \times 10^6 MHz$ for both $^{85}$Rb and $^{87}$Rb. To fully understand FIG. 1 we must introduce a little bit of quantum mechanics. As you can see both the $5S_{1/2}$ and $5P_{3/2}$ are split into multiple substates, each labeled with a corresponding total angular momentum of the atom, $F$, for the ground state and $F'$ for the excited state where

$$F = I + J.$$ \hfill (2)

For ground state of $^{85}$Rb the nuclear spin,

$$I = \frac{5}{2},$$ \hfill (3)

and the total angular momentum of the electron,

$$J = \pm \frac{1}{2}.$$ \hfill (4)

Using these values in conjunction with Eq. 2 we can see that we get $F = 2$ and $F = 3$. The process is the same for $^{87}$Rb, except now we have a value of one less for $I$ thus our $F$ states are now $F = 1$ and $F = 2$. In order to obtain $F'$ for $^{85}$Rb and $^{87}$Rb we must now consider that the excited state is now a P orbital instead of an S orbital and thus the allowed values for $J$ are now

$$J = \pm \frac{1}{2}, \pm \frac{3}{2}.$$ \hfill (5)

This will introduce 2 additional hyperfine states that were not present in the ground state splitting and plugging these values into Eq.2 it is clear to see that the values of $F'$ are easily obtained. A semiclassical model that may help with the visualization of different energy levels would be to consider the electrons as magnets with a magnetic moment that is proportional to the orbital angular momentum. This allows us to think of the electron’s magnetic moment as being "aligned" and "antialigned" with the induced magnetic field from the nuclear spin creating two different energy states for each value of orbital angular momentum. In the S state the allowed value of L will be 0 and for the P state allowed values will be 1 and 0. While not technically correct it can be helpful to think of the "strength" of these electron magnets as being determined by the angular momentum and the direction of the magnetic moment as being determined by the spin of the electron. This will result
FIG. 1. An energy diagram of $^{85}\text{Rb}$ and $^{87}\text{Rb}$, showing the hyperfine splitting caused by energy perturbations from magnetic moments created by the nucleus and angular momentum of electrons.

in a higher energy state where the electrons magnetic moment is antialigned with the nuclear B field and a lower energy state where the electrons magnetic moment is aligned with the nuclear B field. The introduction of an additional option for the orbital angular moment now just creates an even higher high energy state and an even lower low energy state.

III. DOPPLER BROADENING

Doppler Broadening is a result of thermal motion of the gas molecules in the Rubidium sample. Since this experiment was not conducted at or near absolute zero the gas molecules of the sample move with velocities that follow a Maxwell distribution, thus an atom moving in the +x direction that encounters a wave also moving in the +x direction will experience the wave as doppler shifted to a lower frequency and an atom moving in the +x direction that encounters a wave moving in the -x direction will experience the wave as being doppler shifted to a higher frequency. The equation that shows the experience frequency is as follows,

$$\omega = \omega_0 (1 - \frac{v}{c}),$$

where $\omega_0$ is the frequency that the atom would experience if it weren’t moving in the x direction at all. This leads to what is known as doppler broadening which is a result of reaching a perceived resonant frequency based on the velocity of the atom with respect to the propagating wave thus instead of getting a single resonant frequency we get a spread of frequencies that can be described as a Gaussian with the form,

$$I(\omega) = I_0 e^{-\frac{m_0 c^2 (\omega_0 - \omega)^2}{2 k T}},$$

from here we know that the full width at half maximum is

$$\Delta \omega = \frac{2 \omega_0}{e \sqrt{2\ln\frac{k T}{m_0}}}$$

IV. EXPERIMENTAL SET-UP

This experiment utilizes what is called Saturation Absorption Spectroscopy. A diagram of the experimental setup can be seen in FIG. 2, but I will attempt to give an accurate description of each component and it’s importance to the outcome of the experiment in what follows. To begin with we need a coherent light source that will be provided by a 780.2 nm tunable laser diode. This light will then travel to our first beam splitter that will split the beam so that one beam goes to our etalon while the other beam heads along towards the rest of the experiment. The beam that goes towards the etalon is a key component of this experiment as the etalon measures changes in frequency. The etalon fails to give the actual frequency of the beam but gives us a change in frequency that can then be used to determine the frequency spacing of energy transitions when coupled with the absorption spectrum. The second beam then heads to another beam splitter which allows 90% one direction, called the "pump" beam, and 10% another direction which is called the "probe" beam. In the first experiment the "pump" beam is sent into the Rubidium gas from the left and the "probe" beam from the right. The "probe" beam passes through the sample to a PIN Detector that then collects data for the absorption spectrum. The "pump" beam is meant to be significantly stronger that the "probe" beam so as to excite the rubidium atoms to the excited states, then when the "probe" beam travels through the sample the excited electrons will "fall" back to the ground state through stimulated emission which will then be picked up by the PIN detector. This setup is designed so to minimize the effect of Doppler Broadening. In order to even begin to resolve the hyperfine structure we must introduce one more experimental trick that consists of introducing a chopper between the second beam splitter and the Rubidium sample coupled with a lock in amplifier. This is known as Phase Sensitive Detection. PSD simply creates a signal that is frequency dependent on the chopper and any noise that is not frequency dependent essentially gets "thrown out" by the lock in amplifier which allows for higher resolution and thus a detection of the hyperfine structure.

V. RESULTS

A. Finding Temperature from Doppler Broadening

In order to check our model I used FIG.3 as well as Eq.8 to determine the temperature at which the experi-
FIG. 2. A drawing of the experimental setup

FIG. 3. A graphic of the 4 principle absorptions for $^{85}\text{Rb}_{a,b}$ and $^{87}\text{Rb}_{a,b}$

ment was conducted and check this value with room temperature. After doing the calculation $T = 344K \pm 45K$ which just barely squeaks by to include room temperature since the discrepancy is within my uncertainty. Errors were introduced through the resolution of the data as well as uncertainty of frequency introduced by the resolution of the etalon. In other words the etalon data was not infinitely sharp thus there was a spread of values that introduced an uncertainty that propagated through the calculations.

B. Frequency Measurements of $^{85}\text{Rb}_{a,b}$ and $^{87}\text{Rb}_{a,b}$

To measure the the change in frequency between corresponding isotope peaks I first needed to establish a base through the etalon data through which I then converted the separation of peaks in the absorption spectrum into frequencies and then just found the difference between them. Results can be found in Table I. Error was introduced through uncertainty of etalon peaks as well as uncertainty of absorption peaks.

TABLE I. Ground State Frequency Differences

<table>
<thead>
<tr>
<th></th>
<th>Expected</th>
<th>Experimental</th>
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<tbody>
<tr>
<td>$^{85}\text{Rb}$</td>
<td>3036 MHz 2850MHz ± 219MHz</td>
<td></td>
</tr>
<tr>
<td>$^{87}\text{Rb}$</td>
<td>6835 MHz 6407MHz ± 493MHz</td>
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FIG. 4. A graphic of the hyperfine structure for $^{87}\text{Rb}$ showing the hyperfine structure as well as the crossover peaks.

FIG. 5. A graphic of the hyperfine structure for $^{85}\text{Rb}$ clearly showing the lack of resolution necessary to resolve the energy levels of the hyperfine structure.

C. Frequency Measurements of Hyperfine Spectrum for $^{85}\text{Rb}$ and $^{87}\text{Rb}$

The hyperfine spectrum found for $^{87}\text{Rb}$ was conclusive with theory and a collection of the results can be found in Table II, as well as a graphic of the hyperfine structure (FIG.4). Again, uncertainty was introduced through the propagation of uncertainty in the etalon peaks as well as the uncertainty introduced through the determination of individual hyperfine peaks. The resolution for this experimental setup is not enough to differentiate the hyperfine structure of $^{85}\text{Rb}$ and a clear representation of this can be found in FIG.5.

TABLE II. $^{87}\text{Rb}$ Hyperfine Frequencies

<table>
<thead>
<tr>
<th></th>
<th>Expected</th>
<th>Experimental</th>
</tr>
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<tbody>
<tr>
<td>$F' = 1 \rightarrow 2$</td>
<td>157 MHz 158MHz ± 15MHz</td>
<td></td>
</tr>
<tr>
<td>$F' = 2 \rightarrow 3$</td>
<td>287 MHz 270MHz ± 25MHz</td>
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VI. CONCLUSIONS

All results aforementioned are conclusive with theory as the discrepancy of the results falls within the uncertainty of the data. The resolution of this experiment
setup is enough to resolve the hyperfine structure of $^{87}\text{Rb}$ 
but fails to resolve the hyperfine structure of $^{85}\text{Rb}$. The 
ground state energies can also be resolved using the experimental techniques 
described as well as the doppler broadening.

VII. REFERENCES