An exploration into Zeeman Splitting and Rabi Oscillations

Kjell Hiniker, John Prior[1]
University of San Diego

(Dated: 15 December 2015)

In this experiment we aim to find the g_f values and the quantum number I for both ^{85}Rb and ^{87}Rb as well as confirm the Quadratic Zeeman Splitting associated with the Breit-Rabi equation. Measured values for g_f were $.31 \pm .04$ and $.47 \pm .09$ for ^{85}Rb and ^{87}Rb respectively these values are comparable to the values reported in literature of $\frac{1}{3}$ and $\frac{1}{2}$ respectively.. Measured values for I were $2.54 \pm .05$ and $1.54 \pm .1$ for ^{85}Rb and ^{87}Rb respectively. These values are also comparable to values reported in literature which are $\frac{5}{2}$ and $\frac{3}{2}$ respectively. We also confirmed the number of magnetic sub-levels in the ground state for ^{85}Rb and ^{87}Rb through the Quadratic Zeeman Effect.

I. INTRODUCTION

Optical pumping is an important technique used to isolate certain energy levels that can then be probed for information. In conjunction with the Zeeman effect we can explore the quantum number I. Unlike laser spectroscopy with relies on tuning a laser to find a resonant wavelength that then excites electrons to a higher energy level, Optical pumping relies on matching the energy levels to a set wavelength. The two processes can be seen as complementary to each other in that they both aim to measure energy gaps between levels but go about doing so in seemingly opposite ways. The Zeeman Effect is the process of splitting an energy level even further through the presence of an external magnetic field. The energy of these levels is proportional to the applied magnetic field as well as the spin interactions between the nucleus and the electron, thus if either magnetic field or the spins are known we can solve for the other. This made the Zeeman effect an effective tool in establishing and confirming spin quantum numbers for the nuclei of atoms. In the same way that Optical Pumping is connected to laser spectroscopy, the Zeeman effect is related to Nuclear Magnetic Resonance. The spin of the nucleus played an integral role in the PNMR experiment as the spin produced a magnetic moment in the nucleus that we could then perturb with a transverse magnetic field which then allowed us to measure multiple relaxation times. In a way this experiment is somewhat of a synthesization between the PNMR and the Laser Spectroscopy experiments. Section II aims to describe the process of Optical pumping while also trying to make connections to simpler analogies to deepen intuitive understanding. Section III and IV discuss two different types of Zeeman Splitting in relation to this experiment and Section V reviews the set-up of the experiment. Section VI reveals the results of the experiment and Section VII concludes the paper with discussion on the integrity of the experiment.

II. UNDERSTANDING OPTICAL PUMPING

In order to understand this experiment we must first develop an understanding for a certain set of quantum numbers and their influence on various energy levels within the atom. In FIG. 1 a list of each quantum number can be seen as well as there respective definitions. The quantum numbers that we will be most concerned with in this experiment are F, I, and M. M is dependent on an applied external magnetic field that then splits each "F" level of the Hyperfine Structure into $2F+1$ states. These states are dependent on the quantum number, M, and a depiction of this can be seen in FIG. 2. The Hamiltonian responsible for this energy splitting is

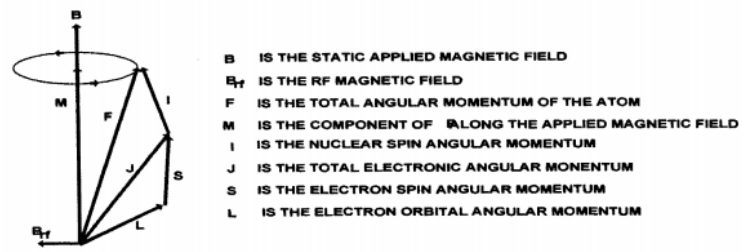

FIG. 1. A depiction representing each quantum number.

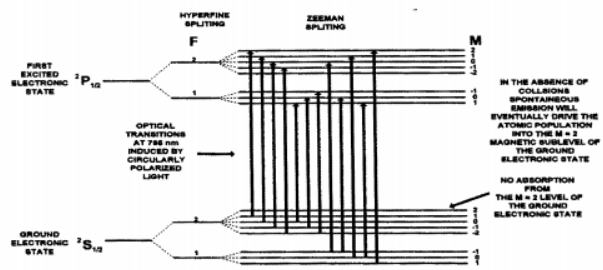

FIG. 2. A picture showing both the Hyperfine splitting and the further splitting caused by the applied external magnetic field.

$$H = haI \bullet J - \frac{\mu_J}{J} J \bullet B - \frac{\mu_I}{I} I \bullet B. \qquad (1)$$

As you can see from the EQ. 1 the Hamiltonian is dependent on I, J, and B which is indicative of the fact that energy splitting is determined be the interaction between the nuclear magnetic moment, I, the electron magnetic moment, J, and the applied external magnetic field B.

The third term of this Hamiltonian is the interaction between the nucleus and the external magnetic field and thus is unnecessary as we are concerned with the electrons' spin and its interaction with the external magnetic field. This begs the question as to why I, the nuclear spin, should even be considered when looking at the electron spins. The short answer is that we still need to consider that the F energy levels are split due to the interaction between the nuclear spin, I, and the spin of the electrons thus in order to fully understand the electron spin we must consider the nuclear spin. The next key concept we must understand is how Optical Pumping actually aims to probe these energy states. To begin I will use a far simpler model to demonstrate a few key ideas and then take those ideas and apply them to the situation that is present in this equation. FIG. 3 shows a very simply the general idea of how optical pumping works. Essentially what is going on here is that "water" is being pumped out of Levels 1 and 2 into Level 3. Level 3 then drains its water into Levels 1,2, and 4 equally. As time progresses the water that ends back in Levels 1 and 2 will continually be pumped back to Level 3 and a small portion of this water will become "stuck" in Level 4 from which it can not be pumped out of. The amount water that Levels 1 and 2 pump becomes less and less and the water in Level 4 increases until all the water eventually ends up in Level 4. This is directly analogous to what is happening in the Rubidium atoms of this experiment. In optical pumping we are simply pumping electrons, the equivalent to water in the analogy, from the M states of ${}^2S_{\frac{1}{2}}$ to the M states of ${}^2P_{\frac{1}{2}}$, but theres a catch, in this experiment the photons that are "pumping" the electrons through absorption is circularly polarized to only allow transitions of $\Delta M + 1$. This restriction on the transitions allows all of the states to be excited to a level in the ${}^2P_{\frac{1}{2}}$ except the $M = +2$ state of the ${}^2S_{\frac{1}{2}}$ level. This is because in order for this state to make a transition there would need to be a $M = +3$ state in the ${}^2P_{\frac{1}{2}}$ level. This $M = +2$ state is the equivalent of Level 4 in the water analogy; in other words all the other states can have photons excited to higher energy levels which eventually drop back down to the lower energy levels but get "stuck" in the $M = +2$ state because any photon in this states can not be excited out of that state by the same pumping mechanism.

A. Transient Effects

We will also take a brief look at some other interesting effects that can be looked at with Optical pumping. In this situation we have essentially created a two level system between $M = 2$ and $M = 1$. When we perturb this system with a transverse magnetic field, much like what we did in NMR, we can create Rabi Oscillations between the states. What we expect to see is that with an increasing strong magnetic field the period of oscillation between the two states should increase. Even intuitively this makes sense because the Magnetic field induced by the RF signal is the magnetic field that is responsible for the precession of F about the the B_{RF} and thus if it is stronger the precession should occur faster resulting in a higher frequency and lower period.

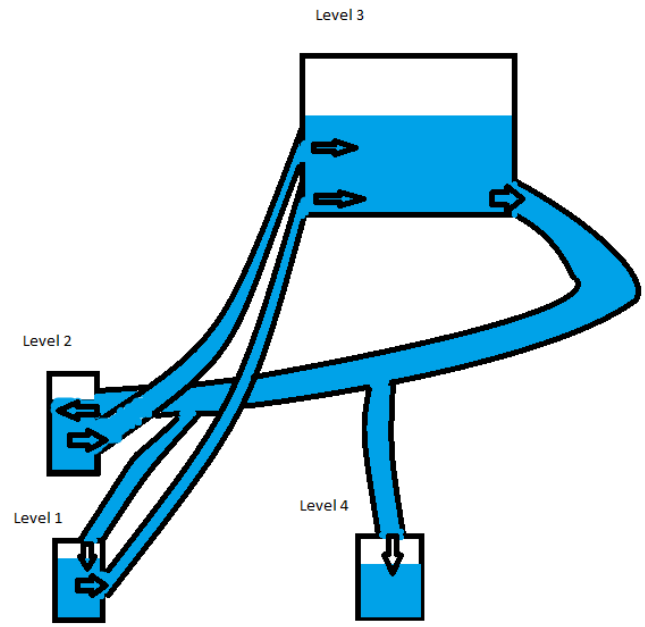

FIG. 3. A simpler version of the same concept used in Optical Pumping.

III. LOW FIELD ZEEMAN SPLITTING

In the first part of this experiment we looked at what is known as the Low Field Zeeman Splitting. The splitting of the energy levels caused by the applied magnetic field can be described using the Breit-Rabi equation which goes as follows,

$$W(F,M) = -\frac{\Delta W}{2(2I+1)} - \frac{\mu_I}{I}BM \pm \frac{\Delta W}{2}\left[1 + \frac{4M}{2I+1}x + x^2\right]^{\frac{1}{2}} \tag{2}$$

where

$$x = (g_f - g_I)\frac{\mu_0 B}{\Delta W}. \tag{3}$$

This equation is dependent on the dimensionless variable x and plot of this relation can be seen in FIG.4. EQ.2 gives rise to three different areas of interest, the first of which is when x is very close to 0. With the condition that x is very close to 0 the differences in energy of the states is linear with B and follows the equation,

$$W = g_f \mu_0 B M. \tag{4}$$

From this equation we can find the values of g_f for ^{85}Rb and ^{87}Rb.

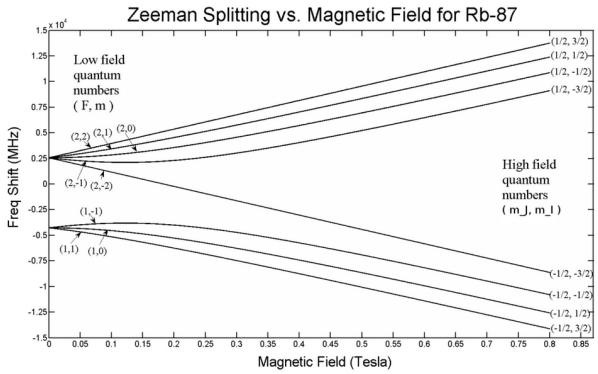

FIG. 4. A plot of the difference in energy levels versus the dimensionless quantity x

IV. QUADRATIC ZEEMAN SPLITTING

A second region can also be examined in this experiment and is defined as the region where $0 < x < 2$. In this region the energy levels are split quadratically instead of linearly. In order to investigate this area we must go to higher values of B. In this region we can see each the transitions associated with each "M" energy level. FIG. 4 shows a plot the energy differences of each level versus the dimensionless quantity, x. The three regions described can easily been seen in FIG.4.

V. TRANSIENT EFFECTS

In this section we decided to see what would happen if, when tuned to resonance, we turned the RF signal on and off rapidly at a frequency of 5Hz. We then varied the voltage of the RF signal to try to find a relationship between the voltage(which in this case is also indicative of the strength of the magnetic field created by the RF signal) and the period of a "ringing" oscillation.

VI. EXPERIMENTAL SET-UP

In this experiment we used the set-up that can be seen in FIG.5. The RF discharge lamp creates light of two main wavelengths, 780nm and 795nm. This light then passes through the interference filter which filters out the 780nm wavelength leaving only the 795nm photons. The remaining light then passes through the linear polarizer and the quarter wave plate which accounts for the restrictions of $\Delta M + 1$ in the energy transitions which makes this whole process possible. We have control over the RF Magnetic field and the static magnetic field which we can tune in accordance with each other to detect transitions. The light is then detected by the optical detector. Essentially what happens is that we start with a population of electrons spread evenly between all states, the light is then turned on which pumps all of the photons

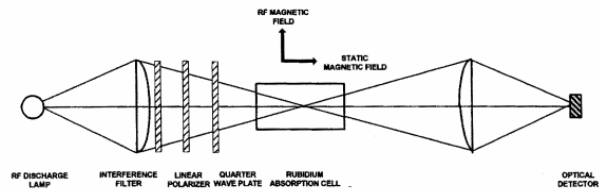

FIG. 5. A picture of the experimental set-up

into one level, namely the $M = +2$ state, then we scan a range through the static magnetic field. This is effectively changing the ΔE between the states. Once the ΔE is equal to the energy that we are producing with the RF magnetic field we will get transitions out of the $M = +2$ state which will be shown as a dip in intensity as the light can now be absorbed by the electrons.

VII. RESULTS

A. Calculating g_f from the Low Field Zeeman Splitting

As seen in EQ.4,

$$W = g_f \mu_0 B M, \tag{5}$$

which means that

$$f = g_f \mu_0 \frac{B}{h}. \tag{6}$$

From this equation we can calculate values of g_f for both ^{85}Rb and ^{87}Rb. By plotting the frequencies of transition against the applied magnetic field B and known values of μ_0 and h we can solve for g_f by finding the slope of the line formed by this plot. A plot of this relationship can be seen in FIG.6. To find the values of g_f we simply need to divide the slope of each line by $\frac{\mu_0}{h}$, resulting in a value of $.47 \pm .09$ for ^{87}Rb and $.31 \pm .04$ for ^{85}Rb. Error was introduced based on the uncertainty of determining the magnetic field at which the transitions occurred which basically means that the frequency at which the transitions occurred has some "width", if you will, that creates some uncertainty. This uncertainty gets carried through the calculations to arrive at the values presented. That being said the discrepancy fell within the bounds of our uncertainty for g_f.

B. Calculation Nuclear Magnetic Spins

In order to calculate the nuclear magnetic spins we first need to know that

$$g_f = g_J \frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)} \tag{7}$$

where

$$g_J = 2.00232. \tag{8}$$

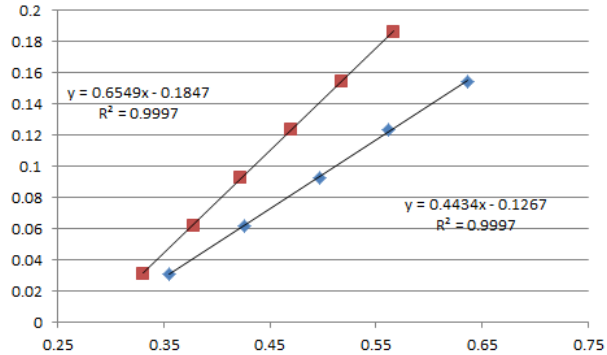

FIG. 6. A plot of B vs f with ^{87}Rb in red and ^{85}Rb in blue. The x-axis is actually in units of current but because the conversion from current to Guass in this case the conversion doesn't change the slope. The y-axis is in units of Mhz.

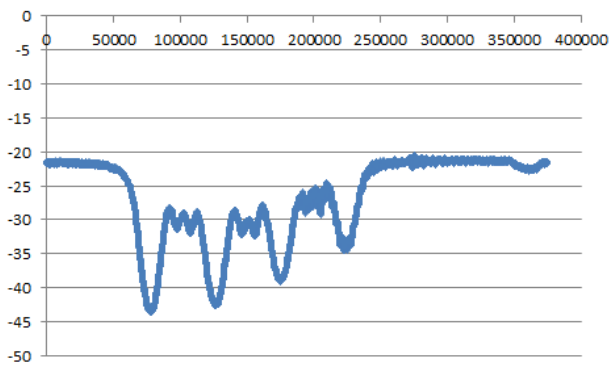

FIG. 7. A plot of ^{87}Rb Quadratic Zeeman Splitting at .016 Gauss and a frequency of 2.85 Mhz. The x-axis simply represents number of data points.

Plugging in values for g_f, F and J we can solve for I. For ^{85}Rb a value of $2.51 \pm .05$ was obtained and for ^{87}Rb a value of $1.54 \pm .01$. Again, the error in the data is within the discrepancy from theory with the theoretical values being $\frac{5}{2}$ and $\frac{3}{2}$ respectively.

C. Quadratic Zeeman Splitting

In this section the only thing we were able to do was to confirm that the number of dips measured matches the expected number of dips for each isotope. In this paper we can confirm that ^{87}Rb is measured to have the 6 dips that theory predicts. Again, the number of dips to be expected comes from the number of M states that are present which is governed by $2F + 1$. ^{87}Rb has 8 magnetic sub-levels which arise from 5 sub-levels from the $F = 2$ state and 3 sub-levels from the $F = 1$ state. This allows for a total of 6 transitions. By the same logic ^{85}Rb should have 10 allowed transitions and thus 10 dips in total. The 6 dips of ^{87}Rb can be seen in FIG. 7.

TABLE I. Ground State Frequency Differences

	Expected	Experimental
^{85}Rb g_f	$\frac{1}{3}$	$.31 \pm .04$
^{85}Rb I	$\frac{5}{2}$	$2.54 \pm .05$
^{87}Rb g_f	$\frac{1}{2}$	$.47 \pm .1$
^{85}Rb I	$\frac{3}{2}$	$1.54 \pm .01$

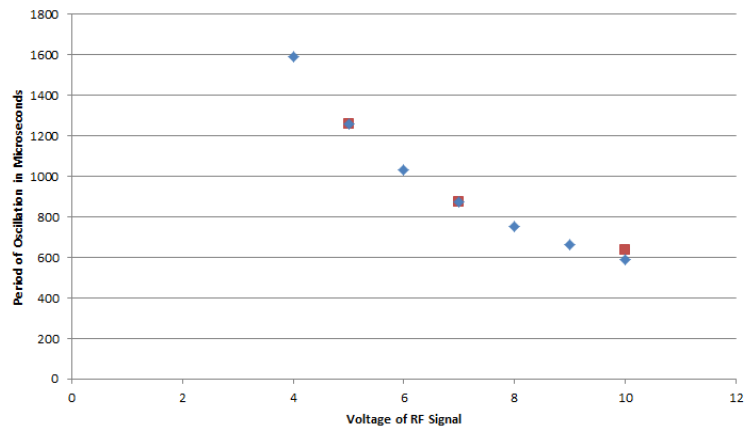

FIG. 8. A graph of both the a theory curve that is proportional to $\frac{1}{V}$ (in blue) and the data collected (in red).

D. Transient Effects

After some analysis it is clear that the period of oscillation between the two states does in fact become shorter the stronger B_{RF} is. In FIG.8 you can see the data collected as well as a theory curve that is proportional to $\frac{1}{V}$. As we can see the period follows the expected relationship with the voltage of the RF signal. FIG.9 shows the actual oscillations between the $M = 2$ and $M = 1$ state.

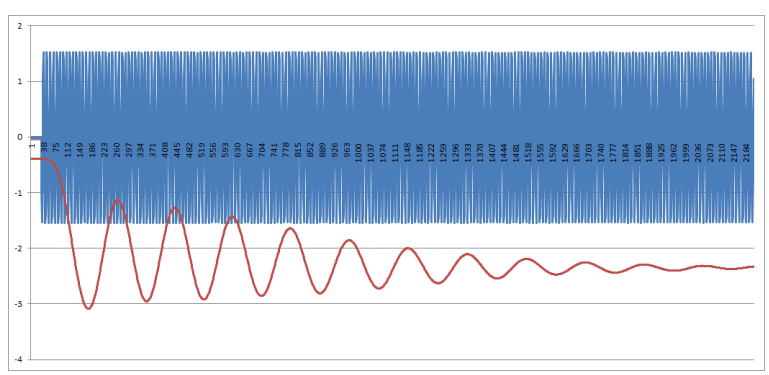

FIG. 9. The blue shows the gating sequence of the resonant RF signal while red shows the data picked up by the photodetector which is essentially showing the flipping of electrons between states.

VIII. CONCLUSIONS

It is clear to see that the data collected matches theory very well. Values calculated for both g_f and the nuclear magnetic spin match almost exactly with that predicted by theory and reported in literature. Errors were introduced in measuring the peaks of the Low Field Zeeman Effects but the discrepancy of the data was well within the uncertainty of the data leading us to conclude that the values calculated can be confirmed to be in agreement with the values cited in literature.

IX. REFERENCES

[1]Teach Spin, Optical Pumping of Rubidium: Instructor?s Manual, OP1-A, (Buffalo,NY, 2002).

[2]D.J Griffiths, *Introduction to Quantum Mechanics*,(Pearson Education Inc., Upper Saddle River, 2005).