Measuring velocity of ion acoustic waves through plasma

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(Dated: 11 December 2015)

This experiment aimed to measure velocity of ion acoustic waves propagating through a fluid medium. The velocity was measured two different ways and then compared to analyze the difference between the two ways. The first measurement of velocity came from the group velocity definition seen in Equation 17 and the second measurement of the velocity came from electron temperature from the Langmuir probe and the definition of thermal velocity seen in Equation 1. Velocities were measured at two different pressures. At a pressure of 9.5×10^{-4} Torr, the group velocity was 1941 m/s ± 10 m/s and the thermal velocity was 4.04×10^5 m/s $\pm 10^3$ m/s. At a pressure of 2.5×10^{-4} Torr, the group velocity was 2764 m/s ± 10 m/s and the thermal velocity was 4.44×10^5 m/s $\pm 10^3$ m/s.

I. INTRODUCTION

Plasma is a fourth state of matter. Very simply put, it is an ionized gas which has enough energy to allow ions and free electrons to exist. A plasma is a gas that is energized enough enough such that electrons break away from atoms to create loose ions. Plasma is the most common state of matter in the universe. In fact most stars are simply just giant balls of plasma. Our own sun is a plasma energized by enough heat that the electrons are stripped away from the hydrogen and helium atoms that make up our sun.

Plasma physics is defined as the study of charged particles and fluids interacting with electric or magnetic fields. The study of plasma has many applications in the fields of astrophysics, controlled fusion, accelerator physics, beam storage, and more.

The term plasma was coined by Czech medical scientist Johannes Purkinje when referring to blood. Irving Langmuir, reminded of the way blood flowed, used the term plasma to describe an ionized gas in 1927. Langmuir and his colleague Tonks developed the theory of plasma sheaths, which are boundary layers between ionized plasmas and solid surfaces, when trying to extend the lifetime of Tungsten filament bulbs. The pioneer behind connecting plasma physics with astrophysical phenomena was Hannes Alfven. He developed the theory of magnetohydrodynamics (MHD); MHD treats plasma as a conducting fluid. MHD has been successfully used to study sun spots, solar wind, star formation, and other topics. Studying plasma has also been heavily researched for the development of weapons and energy sources[1] [2].

The overarching goal of this experiment was to familiarize us with plasma, its properties, and how to make measurements of the plasma. For this particular experiment we wanted to measure the velocity of ion acoustic waves propagating through the plasma. To do this we employed two techniques and compared the answers. The first technique involved making measurements of distance and time and plotting those results and then using the definition of group velocity. The second technique involved taking measurements with the Langmuir probe and then using the definition of thermal velocity to calculate velocity. Both techniques and their results are explained in detail in later sections of this paper.

The second section of this paper will cover the theory necessary to understand the procedure, results, and observations from the plasma experiment. Section III is an overview of the experimental design and apparatus used. Section IV further details the experimental procedure and the results of the experiment. This section includes a discussion of the various equations presented in the theory section and how they were manipulated and used to derive the results from the recorded data. Additionally there is an explanation of experimental error and uncertainty associated the results. Section V is a conclusion that compares the two sets of results and talks about the overall usefulness of plasma research.

II. THEORY

This experiment was divided into two main sections; the first section is making measurements using Langmuir probes and the second part was using the definition of group velocity. Both sections of the experiment aimed to measure ion acoustic wave velocity. In order to best understand the procedure and data analysis it is necessary to first gain a strong foundation in the basic physics of plasma.

A. Basic Plasma Physics

Like stated in the introduction, plasmas are essentially just really hot collections of ions, electrons, and neutral atoms. Plasmas exhibit a medium-like behavior. In a plasma, electric fields arise that keep the electrons in and push the ions out. This leaves the plasma essentially neutral. The electrostatic potential structure that arises is called a plasma sheath. The plasma sheath is the result of Debye shielding. A plasma sheath is the layer in a plasma that has a greater density of positively charged ions. This positively charged layer balances the negatively charged surface that the plasma is in contact with.

Plasma sheath's arise because the electrons in a plasma are much lighter than the ions. The temperature of the

electron is also generally hotter than the ion temperature. The velocity of a particle is given by

$$V_{th} = \sqrt{\frac{KT}{M}}. \tag{1}$$

Using the equation it is evident that the electrons are moving faster than the ions. Therefore initially the electrons will be flying all over the place charging the surface they are contained in negative compared to the plasma. Then due to the Debye shielding and increasing electrostatic potential eventually an equilibrium is reached. In this equilibrium the ions have formed a layer around the electrons and neutral atoms. The ions get even more energy in the plasma sheath than they did when forming. The kinetic energy gained in the sheath has an enormous variety of applications.

B. Physics of Langmuir Probe Measurements

Langmuir probes are used in low temperature plasmas to measure density, electron temperature, and the plasma potential. To do this current-voltage characteristic of the probe and the applied bias voltage are obtained.

When the bias voltage, V_B, is sufficiently negative compared to the plasma potential, V_p, the probe collects the ion saturation current, I_i^*. Ion saturation current is given by

$$I_i^* = \frac{1}{4} n_i e v_{th,i} A_p, \tag{2}$$

where n_i is ion density and $v_{th,i}$ is thermal velocity of the ions.

When the bias voltage, V_B is much greater than plasma potential, V_p, the probe collects the electron saturation current. Electron saturation current is given by

$$I_e^* = \frac{1}{4} n_e e v_{th,e} A_p, \tag{3}$$

where n_e is electron density and $v_{th,e}$ is thermal velocity of the electrons.

Overall the current of the plasma can be approximated as

$$I_p \approx I_i^* - I_e^* \exp e(v - v_s)/KT_e. \tag{4}$$

The first term in Equation 4 can be called the ion branch and the second term can be called the electron branch[3].

The floating potential of the plasma, V_f, is given by

$$I(V_f) = 0. \tag{5}$$

The floating potential is defined as the voltage at which there is no net current from the plasma. At the floating potential the plasma potential rises and falls to whatever potential is necessary to maintain zero net current. The plasma potential, V_p is given by

$$I"(V_p) = 0. \tag{6}$$

The plasma potential is defined as the maximum slope on the current-voltage characteristic.

The floating potential and the plasma potential in conjunction with Equation 4 can be combined to create an equation to solve for the temperature of the electrons in the plasma. This equation is given by

$$\frac{KT_e}{e} = \frac{2(V_p - V_f)}{ln\frac{(2M_i}{\pi m_e}}. \tag{7}$$

C. IAW Physics

Acoustic waves exist if the wavelength of the mean free path of the medium is much much shorter than the wavelength of the sound. The wavelength of mean free path is given by

$$\lambda_{mfp} = \frac{1}{n\sigma}. \tag{8}$$

A good approximation to remember when working with plasma physics and vacuum experiments is that

$$\lambda_{mfp} \approx \frac{5cm}{P(mTorr)}. \tag{9}$$

Acoustic waves exist in plasma. The waves studied in this experiment were called ion acoustic waves. These are low frequency waves that result from introducing a perturbation into the plasma; which thereafter propagates throughout the plasma. The goal of this experiment is to measure the speed of the wave propagation using both Langmuir probe measurements and the definition of group velocity.

The perturbation to the ion density equilibrium is described by three equations. The first equation is from Newton's second law and is given by

$$Mn_i \frac{\partial v}{\partial t} + v\frac{dv}{dx} = en_i\vec{E} + \vec{V_x} \times \vec{B}. \tag{10}$$

The second equation is a continuity equation given by

$$\frac{\partial n}{\partial t} + \frac{d}{dx}(vn_i) = 0. \tag{11}$$

The third equation is Poisson's equation which is given by

$$\nabla^2\phi = \frac{\rho}{\epsilon_0} = (en_i - en_e)/\epsilon_0. \tag{12}$$

From these equations a dispersion relation was derived:

$$\omega^2 = \frac{C_s^2 K^2}{(1 + K^2\lambda_D^2)}, \tag{13}$$

where λ_D is the Debye wavelength and c_s is the phase velocity of ion acoustic waves. The Debye wavelength is given by

$$\lambda_D = \sqrt{\frac{\epsilon_0 KT_e}{n_0 e^2}}. \tag{14}$$

The phase velocity is given by

$$c_s = \lambda_D \Omega_p = \sqrt{\frac{KT_e}{M}}, \tag{15}$$

where Ω_p is the ion plasma frequency. Ion plasma frequency is given by

$$\Omega_p = \sqrt{\frac{n_o e^2}{\epsilon_0 M}}. \tag{16}$$

Plasmas don't like to propagate waves with the wavelength less than the Debye wavelength. The data looked at for this experiment was on a plot of frequency as a function of k when $k\lambda_D < 1$ which can be called the big wavelength limit. When $k\lambda_D < 1$ the group velocity and the phase velocity are equal. Group velocity, the speed at which waves convey signals is given by

$$v_g = \frac{d\omega}{dk}. \tag{17}$$

Phase velocity is simply the speed of the wave is given by

$$v_\phi = \omega/k.\ ^4 \tag{18}$$

III. EXPERIMENTAL SETUP

The block diagram in Figure 1 shows a basic version of the experimental setup. The experimental setup was

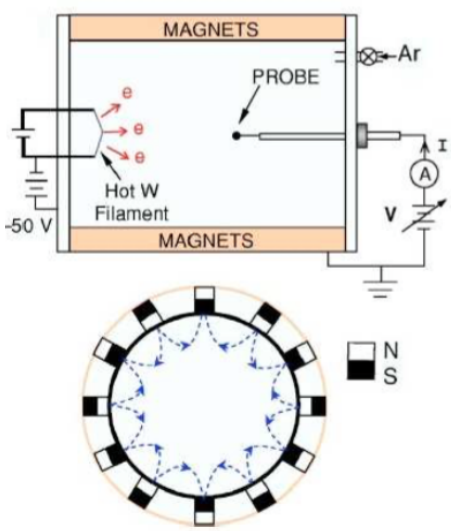

FIG. 1. Schematic diagram showing the experimental setup of the plasma experiment. This block diagram only shows the basic setup of the chamber. It does not include the accessory components needed to make the plasma, record data, and create acoustic waves

designed to allow us to make Langmuir probe measurements and group velocity of ion acoustic waves propagating through the plasma. The super-duper turbo pump creates extremely low pressures within the chamber. The pump is always left running to ensure that there is always a good base vacuum within the chamber. The tungsten filament within the chamber is heated up to thermionically emit electrons. The magnets around the chamber are used to repel the electrons inside the plasma. This allows for better containment of the plasma and plasma sheath formation. In addition to the Langmuir probe in the chamber, there is a wave launching grid. The wave launching grid, controlled by an external waveform generator, perturbs the plasma to create ion acoustic waves that propagate through the plasma.

THe plasma inside the chamber is made according to the equation

$$e_p^- + A \rightarrow A^+ + e^- + e^-. \tag{19}$$

This equation shows how the electrons thermionically emitted from the tungsten filament collide with the neutral Argon atom to create an argon ion and an additional electron that make up the plasma.

IV. RESULTS

The two main goals of this experiment were to teach us how to take and interpret Langmuir probe measurements and to make measurements of the ion acoustic wave group velocity. The Langmuir probe measurements were used to make T_e measurements to make a $v_t h$ calculation using Equation 1. The other method of measurement used to calculate the ion acoustic wave group velocity was made using the definition of group velocity given by Equation 17. The two velocities were then analyzed and compared. The data was taken for both a high pressure of 9.6×10^{-4} Torr and a low pressure of 2.5×10^{-4} Torr.

A. Langmuir Probe Measurements

The Langmuir probe measurements yielded a current-voltage characteristic plot like previously discussed in the theory section. The data collected from the Langmuir probe yielded both current and voltage data. When plotted as current on the y-axis and voltage on the x-axis, a current-voltage characteristic was created. Figure 2 is the current-voltage characteristic for the plasma at a pressure of 9.6×10^{-4} Torr.

Figure 3 is the current-voltage characteristic for the plasma at a pressure of 2.5×10^{-4} Torr.

In both Figure 2 and Figure 3 it is evident that there was a positive ion branch and a negative ion branch. To be able to make a calculation of the electronic thermal velocity, the first step was to subtract off the ion branch from the data and create a semi-log plot of the electron branch. With the semi-log plot it is possible to determine thermal velocity using Equation 4.

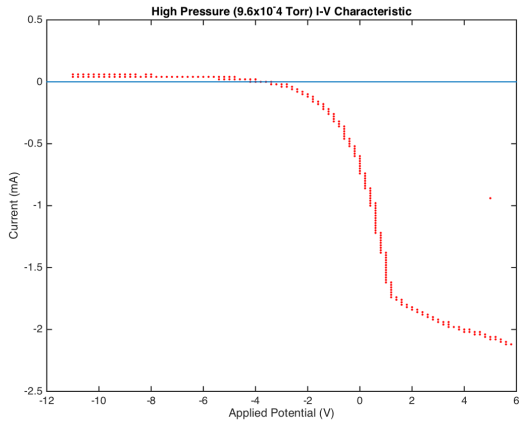

FIG. 2. This is the current-voltage characteristic for the plasma at a pressure of 9.6×10^{-4} Torr. The horizontal segment at the top can be considered the ion branch and the flattening out segment at the bottom can be considered the electron branch according to Equation 4.

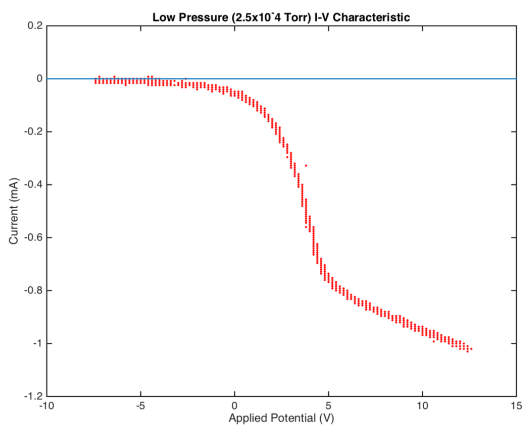

FIG. 3. This is the current-voltage characteristic for the plasma at a pressure of 2.5×10^{-4} Torr. The horizontal segment at the top can be considered the ion branch and the flattening out segment at the bottom can be considered the electron branch according to Equation 4.

At the high pressure of 9.6×10^{-4} Torr the floating voltage was determined to be -4.0 V $\pm$ 0.2 V and the plasma potential was determined to be 0.5 V $\pm$ 0.2 V. Using Equation 7, KT_e was calculated to be 1.01 eV. With this value of KT_e, using Equation 1, V_{th} was calculated to be 4.04×10^5 m/s $\pm$ 1000 m/s. The source of uncertainty in this measurement comes from the human error of determining the floating and plasma potentials on the Matlab plots.

At the low pressure of 2.5×10^{-4} Torr the floating voltage was determined to be -2 V $\pm$ 0.2 V and the plasma potential was determined to be 3.6 V $\pm$ 0.2 V. Using Equation 7, KT_e was calculated to be 1.01 eV. With this value of KT_e, using Equation 1, V_{th} was calculated to be 4.44×10^5 m/s $\pm$ 1000 m/s. The source of uncertainty in this measurement comes from the human error of determining the floating and plasma potentials on the Matlab plots.

A summary of these results is given in Table I.

TABLE I. V_th Results

	V_f	V_p	KT_e	V_th
High Pressure	-4 V	0.5 V	0.9 eV	4.04×10^5 m/s $\pm 10^3$ m/s
Low Pressure	-2 V	3.6 V	1.3 eV	4.44×10^5 m/s $\pm 10^3$ m/s

B. Group Velocity Measurements

The method used to measure the ion acoustic wave group velocity was called the tone burst method. We could not just simply measure the time it took to travel the distance from the grid to the probe directly because the time scales were so small because the electrons were moving too fast to measure on the oscilloscope. The tone burst pulse method involves capacitatively coupled the agilent function generator to the wave launching grid. Figure 4 shows a schematic diagram of the setup of the tone burst method and a drawing of what the oscilloscope would show. The Langmuir probe, in this method, is

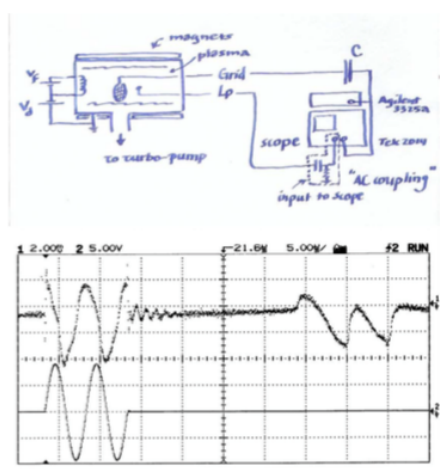

FIG. 4. Schematic diagram showing the experimental setup of the tone burst method for plasma experiment. This block diagram only shows the basic setup of the tone burst method. The bottom section of the figure shows a drawing of what the oscilloscope would record.

used as a detector that is coupled to the ground. An excitation frequency of 250 KHz was chosen because it was above the cutoff frequency but still well below the ion plasma frequency. The time delay between the received pulse and the signal depends on the speed of the waves through the medium. As the Langmuir probe was moved farther away from the grid, there was a change in delay time. The distance and delay time was plotted and it

was revealed that the trend was linear. The slope of the line gives us the group velocity.

The first set of data was taken at a pressure of 9.6×10^{-4} Torr. The uncertainty in the pressure came from the flux of the ions and electrons within the plasma; as the plasma was moving around quickly the the pressure measurement device was trying to keep up with the slight pressure changes. The probe was moved away from the grid a total of 2.5 cm in six 0.5 cm increments. The time delay of the signal was measured at each of these increments. The distance was plotted as a function of time as seen in Figure 5. The slope of the line, which is equal

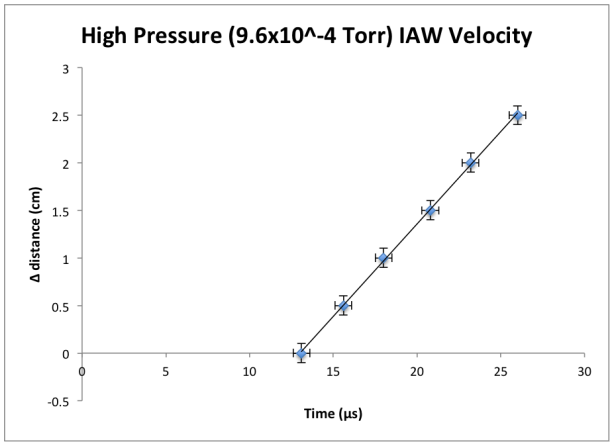

FIG. 5. Plot showing the distance as a function of time. The slope of this plot was equal to the group velocity of the ion acoustic wave.

to the group velocity of the ion acoustic waves propagating though the plasma was calculated to be 1941 m/s $\pm$ 10 m/s. The uncertainty in the measurement stems from human error in measuring the change in distance of the probe to grid distance. The uncertainty also stems from the noise of the oscilloscope making it difficult to place cursors on the same section of the wave for the two wave forms.

The second set of data was taken at a pressure of 2.5×10^{-4} Torr. Once again the uncertainty in the pressure recording came from the measurement device trying to keep up with the changing pressures as a result of the movement of the ions and electrons within the plasma. The probe was moved away from the grid a total of 2.5 cm in six 0.5 cm increments. The time delay of the signal was measured at each of these increments. The distance was plotted as a function of time as seen in Figure 6. The slope of the line, which is equal to the group velocity of the ion acoustic waves propagating though the plasma was calculated to be 2764 m/s $\pm$ 10 m/s. The source of uncertainty in the velocity measurement, just as in the low pressure measurement, stems from human error of distance measurement and oscilloscope noise making the time measurement difficult.

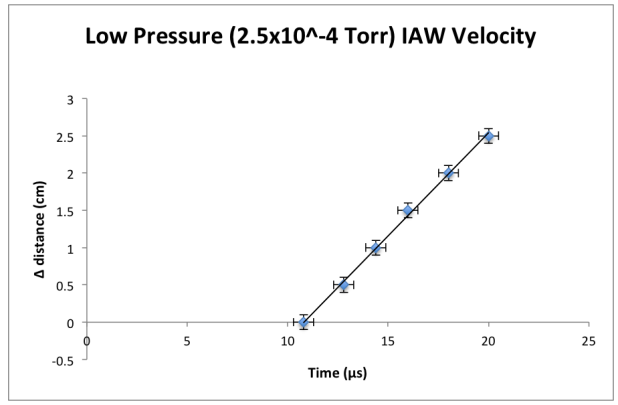

FIG. 6. Plot showing the distance as a function of time. The slope of this plot was equal to the group velocity of the ion acoustic wave.

V. CONCLUSION

The first conclusion we looked to make was to explain the differences in velocity between the two pressures. In the plasma, as the ions, electrons and neutral atoms are moving around at high speeds, there are bound to be collisions. Since both measurements aimed to look at the velocity of the electrons in the plasma that is the point of view taken in the subsequent explanation. The ions and neutral atoms are much more massive than the electron based on the fact that a proton mass is already four orders of magnitude greater than an electron and Argon has 19 protons. So collisions between electrons and neutral atoms and ions yield a large change in velocity for the electron and a small change in velocity of the argon atom. It can be though of as an electron having an elastic collision with a stationary ion or atom. By conservation of linear momentum the velocity of the electron is much slower and the ion or atom increases in velocity only by a small amount.

Based on the logic in the previous paragraph, a more dense plasma equates to an increased rate of collisions. Therefore it makes logical sense that the ion acoustic waves propagate through the less dense plasma with a velocity greater than the more dense plasma.

The second conclusion we looked to make was an explanation of the differences in thermal velocity and group velocity. Table II recaps the results of the thermal and group velocities for both pressures.

TABLE II. Results Summary

Pressure (Torr)	Thermal Velocity (m/s)	Group Velocity (m/s)
9.6×10^{-4}	$4.04 \times 10^5 \pm 1000$	1941 $\pm$10
2.5×10^{-4}	$4.44 \times 10^5 \pm 1000$	2764 $\pm$10

Based on the the results it is obvious that the group velocity is two orders of magnitude smaller than the thermal velocity. This difference is explained by the Bohm

criterion. The Bohm criterion states that the thermal velocity is greater than or equal to the group velocity. This intuitively makes sense in the realm of wave properties. The thermal velocity is just the movement of electrons while group velocity is the speed at which an actual signal is conveyed through that wave.

VI. ACKNOWLEDGEMENTS

I would like to thank Dr. Severn for his help with understanding the background theory and for providing the necessary equipment and assistance with the experimental setup. Additionally, I would like to thank my lab partners, Warren Mardoum and Adam Egbert for helping take and analyze data.

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