Tutorial Questions

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(Dated: Sp 2021)

The process of experimental research involves integrating a great deal of different sorts of understandings. The researcher tries to put together an understanding of physical theory, concepts and ideas, along with a grasp of the capabilities of instruments and techniques in order to arrive at an *experimental design suitable for testing a theoretical model.* You have to understand how oscilloscopes, coax cables, DMM's, LabView vi's, Matlab.m files, photocells, etalons, quarter wave-plates, linear polarizers, and so on, *work*, as surely as you have to understand how perturbation theory in quantum mechanics works. You must perturb Hamiltonians using linear algebra as confidently as you perturb atoms with RF electromagnetic radiation. To help the student weld together a comprehensive understanding of each experiment, we will have tutorial sessions each week of each experiment. Each experiment will come with a set of readings and questions (below!) the answers to which will be prepared in advance before the weekly tutorial meeting. Each experiment will give rise to different specific questions in addition to the common ones. The questions beyond the first 4 are listed in no particular order. Rather it is in the convergence and mutual coherence of (finding) their answers that (a) a clearer picture of the experiment arises, (b) an understanding of how research such as this can be done, and (c) how scientific questions can be posed and answers sought.

I. OPTICAL PUMPING

1. What is the experiment designed to measure? What are the principal results (giving figures of merit, with units, where possible)?

2. How is the experiment designed to measure those quantities?

3. Sketch a block diagram of the apparatus and label all the principal parts. What does each part do?

4. What difficulties are encountered typically (physical, technological, and so on), and how does the design permit these things to be overcome?

5. A quantum refresher. Consider a neutral hydrogen atom. Its n=2 ^2P state is $(2S+1)(2L+1)$ fold degenerate, or 6 fold. Without taking electron spin into account, all these states have the same energy. But the 'spin-orbit' effect removes (some) of the degeneracy. Recall the quantum rules for adding angular momenta, the eigenfunctions and eigenvalues associated with generic angular momentum operators $\hat{J}^2$, and $\hat{J}_z$, and the total angular momenta associated with ^2P electrons, namely, $\vec{J} = \vec{L} + \vec{S}$. I assume you know *the quantum numbers* associated with these quantized angular momenta for the electrons in question. With all this in mind,

   (a) show by direct calculation that the matrix representation of the spin-orbit perturbation,

   $$V_{so} = A\hat{L}\cdot\hat{S} = \begin{bmatrix} \frac{A}{2}\hbar^2 & 0 \\ 0 & -A\hbar^2 \end{bmatrix},$$

   where A is a constant determined by theory.

   (b) Given the value of the fine structure splitting between $^2P_{3/2}$ and $^2P_{1/2}$ terms in hydrogen (look this up on NISTs database of atomic energy levels!), what are the units and magnitude of A?

   (c) Further, what does it mean to say that J, L, and S are good quantum numbers, but m_L and m_S are not? Note, the basis to take to achieve the result above of course is the $|LSJM_J\rangle$ basis, with $J = 3/2$ and $J = 1/2$. Note that the perturbation does not involve the $\hat{J}_z$ operator (or measurement). How degenerate, or what is the statistical weight, each $|LSJM_J\rangle$ state, say, where $L = 1$, $S = 1/2$, and $J = 3/2$. How about where $J = 1/2$?

   (d) What spectral resolution is required to 'resolve' electric dipole transitions to the ground state from these two states. Are such transitions allowed? Are electric dipole transitions allowed *between* these states? If not, are there any?

6. we have recently discussed the design of the Stern-Gerlach experiment. Please compose a few sentences that outline the logic of the experiment, as an experiment that attempts to measure the 'spin' of the electron. Recognize that the context of discover was one in which space quantization was doubted, and half integer quantum numbers unknown. In papers and in tutorials, there is value in making classical arguments regarding how instruments and measurements work, even when they are put to use to exhibit quantum phenomena.

7. What is 'optical pumping'? Can you state what it is simply? Can you account for all the basic features of figure 1 shown below? Explain carefully this figure demonstrates optical, or is reflective of, optical pumping.

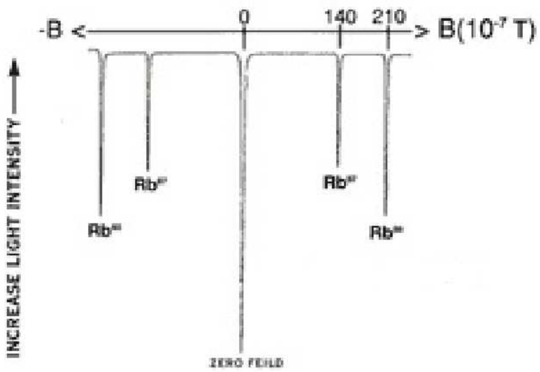

FIG. 1. this plot of detector signal vs. magnetic field demonstrates optical pumping for zero and very low magnetic fields. Both the Rb^{85} and Rb^{87} isotopes produce only unresolved single lines in these low fields. Which dip is which, and according to theory, what must be the frequency of excitation to produce these dips?

8. Where in Quantum Mechanics does one turn to understand what quantum numbers are 'good'? How does this concept come up in this experiment?

9. How are selections rules are determined in Quantum Mechanics?

10. What do the linear polarizer and the quarter wave plates do? Why are they needed? Why are they placed in their particular order?

11. What selection rules govern the 2 kinds of excitation that are crucial to this experiment? What selection rules govern the 2 kinds of decay?

12. Is it necessary to align the optical axis of the detection system with the local magnetic field? If so, why? There are two sets of Helmholtz coils. What do they do? Why are there not 3? How is the magnetic field in this experiment varied? Which?

13. What does the RF energy do?

14. Is it important that the RF coil axis is orthogonal to the other two Helmholtz coils?

15. The light coming from the Rb lamp is spread in wavelength. How much, and does its line width matter, and if so, how? What Doppler shift do we expect in the line coming from the oven?

16. Why is absorption possible only if the Zeeman Effect is zero, if the Rb sample is not bathed in RF?

17. Why is it possible, if RF irradiates the sample at some fixed frequency for there to be a very well defined non-zero external magnetic field for which a very sharp absorption peak is possible? What sets the sharpness or line shape of this peak?

18. what is the spectral resolution of this method? Why was it so celebrated when it was discovered in the '50's?

19. Plot, qualitatively and correctly, the Breit-Rabi diagram for the 2 isotopes, distinguishing one from the other. One of them is shown in Figure 2B-3 in the manual. Describe their principal features.

    (a) Annotate the figures to describe the experimental results. Explain, given these figures how to understand figures 4C-1 and 4C-4 in the manual.

    (b) What resonances are exploited to observe all the dips observed in 4C-1 and 4C-4? What selection rules apply, if any. How is the apparatus designed to produce them, and what can we learn about the desired outcomes from them? What do the little squiggles tell us about the quantum number I for each isotope.

20. What is an isotope shift?

21. What spectroscopic term denotes the ground state energy level? Can one use LS coupling? How does LS coupling determine the term designation?

22. How does nuclear magnetism mess this term designation up? How does it do so physically, and what new quantum numbers are conventionally used to describe the new state(s)?

23. In the 'Conceptual Tour of Optical Pumping', furnished by the vendor of the optical pumping apparatus we use in this experiments, the authors write, 'Both isotopes of rubidium are present in our lamp and sample cell, each with it's own total spin and magnetic moment.' To what spin do they refer? What quantum number is it? F? I? J? S? Explain.

24. Draw an energy level diagram in your lab notebook, with 3 levels of perturbation**s** (3 orders of magnitude for perturbing Hamiltonians, say). The zeroth order include**s** simply the separated Russell-Saunders spectroscopic terms for the ground and excited states. Show the splittings due to a perturbing uniform magnetic field (Zeeman effect) and a perturbing nuclear magnetic moment (Hyperfine splitting). Illustrate what effects these two perturbations have on the spectroscopic terms, and show what happens to resonance radiation absorbed from resonance radiation (why is it called that?) in the $m_F = 0$ states. Show all four types of transitions that can in principle occur (see question # 2 above). Then

    (a) explain qualitatively whether the **intensity of the** absorption transitions **for the case of a weak 'perturbing' magnetic field** are equal to, greater than, or less than, the unperturbed transition (unperturbed by external magnetic fields ~~or by nuclear magnetism~~), the transition that the interference filter is meant to pass. If the absorption requires a photon that is greater or less energetic than the one without perturbations, is it possible to pass the interference filter? What physical 'details' matter here?

(b) perform (theoretical) calculations to support your qualitative explanation

25. How can optical pumping be used to determine g_F and in turn, I? What is g_F, and how big are the nuclear magnetic moment of the nuclei in question? Compare these values to other famous 'g-factors'? What is the significance of their values? It might help to read Benumof's Am. J. Phys. paper. Coming back to g_F, workout from first principles (namely, Quantum Mechanics) how one goes from equation 2B-5 to 2B-7, as displayed in the Optical Pumping manual. Explain the apparent paradox in the phrase introducing 2B-6, "If the interaction with the nucleus is considered, the g factor is given by...", and the explanatory phrase following 2B-7, "where the direct interaction of the nuclear moment with the magnetic field is being neglected."

26. If there were no Helmholtz coils, would it be possible to use the apparatus as a very sensitive magnetometer?

27. Without the RF, would there still be a resonance observable? Is it infinitely sharp? What determines it's width? What would 'the measurement' tell you?

II. PULSED NUCLEAR MAGNETIC RESONANCE

1. What is the experiment designed to measure? What are the principal results (giving figures of merit, with units, where possible)?

2. How is the experiment designed to measure those quantities?

3. Sketch a block diagram of the apparatus and label all the principal parts. What does each part do? (see particularly questions #8 & #9).

4. What difficulties are encountered typically (physical, technological, and so on), and how does the design permit these things to be overcome?

5. A quantum refresher. Consider a neutral hydrogen atom. Its n=2 ^2P state is $(2S+1)(2L+1)$ fold degenerate, or 6 fold. Without taking electron spin into account, all these states have the same energy. But the 'spin-orbit' effect removes (some) of the degeneracy. Recall the quantum rules for adding angular momenta, the eigenfunctions and eigenvalues associated with generic angular momentum operators $\hat{J}^2$, and $\hat{J}_z$, and the total angular momenta associated with ^2P electrons, namely, $\vec{J} = \vec{L} + \vec{S}$. I assume you know *the quantum numbers* associated with these quantized angular momenta for the electrons in question. With all this in mind,

   (a) show by direct calculation that the matrix representation of the spin-orbit perturbation,

   $$V_{so} = A\hat{L} \cdot \hat{S} = \begin{bmatrix} \frac{A}{2}\hbar^2 & 0 \\ 0 & -A\hbar^2 \end{bmatrix},$$

   where A is a constant determined by theory.

   (b) Given the value of the fine structure splitting between $^2P_{3/2}$ and $^2P_{1/2}$ terms in hydrogen (look this up on NISTs database of atomic energy levels!), what are the units and magnitude of A?

   (c) Further, what does it mean to say that J, L, and S are good quantum numbers, but m_L and m_S are not? Note, the basis to take to achieve the result above of course is the $|LSJM_J\rangle$ basis, with $J = 3/2$ and $J = 1/2$. Note that the perturbation does not involve the $\hat{J}_z$ operator (or measurement). How degenerate, or what is the statistical weight, each $|LSJM_J\rangle$ state, say, where $L = 1$, $S = 1/2$, and $J = 3/2$. How about where $J = 1/2$?

   (d) What spectral resolution is required to 'resolve' electric dipole transitions to the ground state from these two states. Are such transitions allowed? Are electric dipole transitions allowed *between* these states? If not, are there any?

6. Imagine a silver atom hurtling through the gap between the pole faces of an extended and extremely spatially homogeneous magnet. All the air has been pumped out (we need to put the source of atoms and the magnetic in a 'good' vacuum chamber; why?) and because of our curiosity and imagination, we put ourselves in the reference frame of the atom, the sole atom between the pole faces. Don't care about which way is 'up' or which way is 'north' for the moment.

   (a) It's a neutral atom (AgI). Does it have a magnetic moment? Why? How?

   (b) Does it align (is it aligning) under the action of the magnetic field? Why or why not? What is that action? Do you think the answer is one thing if one views the atom as a quantum system, and another thing if viewed classically?

   (c) Are there any external electric fields in the frame? Should we care? How would the action of the magnetic field differ if the silver atoms were 'forced' to have zero average velocity, say, by being confined in a glass cell in a vapor (with a buffer gas like neon or argon, etc. with which it would not readily form chemical bonds, nor, on average, agglomerate, nor strike the glass walls very much either)?

7. we have recently discussed the design of the Stern-Gerlach experiment. Please compose a few sentences that outline the logic of the experiment, as an experiment that attempts to measure the 'spin' of the electron. Recognize that the context of discover was one in which space quantization was doubted, and half integer quantum numbers unknown. In papers and in tutorials, there is value in making classical arguments regarding how instruments and measurements work, even when they are put to use to exhibit quantum phenomena.

8. What is a gyromagnetic ratio? Does it have 'units'? What role did this physical quantity play in the discoveries and development of quantum theory?

9. Figure 1 below is a very nice block diagram of pulsed NMR experiment. How would you modify it to make it an accurate reflection of the current set up for PNMR PS-2 spectrometer? Make a new block diagram, or, preferably, edit it the old one, using graphics software.

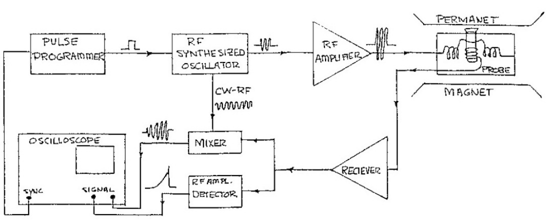

FIG. 2. Compare with figure 2.1 in section II of the PS2 manual and figures 1 and 2 of the conceptual tour to aid in sketching the new version of figure 1a) as it is for the new setup.

10. Again, in the 'Conceptual Tour of TeachSpin's Pulsed NMR', on the first page it is described how the net magnetization is altered by one or more 90^0 or 180^0 RF pulses. The spins, tipped into the x-y plane, then are said to precess around B_o creating a time varying voltage in a pick-up coil. See particularly figures 1 and 2. Critique these statements. How does this make sense from a classical point of view? How does is it to be understood from a quantum point of view?

11. What is free induction decay (FID)? What does learn by observing it? Describe how to use the electronics to measure the so-called free induction decay (FID) of the sample. What relaxation time does the FID signal measure?

12. What is a $\pi/2$ pulse? What is a π pulse?

13. What do the terms 'dephasing' and 'rephasing' mean?

14. Would it be possible to conduct this experiment, as described, at the temperature 'absolute zero'?

15. What is a 'mixed state', from a quantum physics point of view? What is a 'pure state'? Do the experiments described measure one, but not the other? Is the data amenable to a classical description, (apart from the fact that there is space quantization)?

16. What is a 'relaxation time'? Describe the different kinds.

17. In the manual, one finds first order ODE's describing relaxation times, e.g., eq. 1.13 & eq. 1.22, which are versions of the Bloch equation (for this look up D. Ter Haar's paper, *Simple Derivation of the Bloch Equation*, Am. J. Phys. **34,** 1164 (1966)....)

    (a) Consider equation 1.13. Right next to this equation is figure 1.2. (p.16, section I-5). Is this figure the solution to equation 1.13? Under what circumstances? Will the circumstances in the case of our experiments be the same? If so, explain, if not, explain the difference, and amend the model assumptions so that they conform to the experimental results you hope to get.

    (b) Look carefully at figure 8 on page 4 of the manual (referring, as above, to PNMR Manual Part I). Please explain every detail of this. How will this help find T_1 for each of your samples? Will your data look like this? Explain.

18. Write out your own derivation of eq.'s 1-19 and 1-20 in the manual. The problem is one in which there is magnetic moment immersed in a homogeneous and constant magnetic field (B_o), subject to a time dependent magnetic field ($B_1(t)$) of frequency ω in the plane perpendicular to the constant magnetic field. It (the moment..) is initially oriented obliquely to $\vec{B_o}$. Do it for the case of a classical magnetic moment. The point here is to come to a deeper understanding of what's going on. Mathematicians do investigations like this too, and for the same purpose, and the process is sometimes called 'heuristics', investigations to discover, and lawyers sometimes call this a 'vetting' procedure. Solve the problem completely for the classical problem. Discuss in detail what phenomenological difference you expect there to be between the classical and quantum version of this same problem. Then solve the problem for the classical case. And it's useful to think of this

frequency as $\omega = \omega_0 + \Delta\omega$. This problem is discussed in a little detail in Shankar, 'Principles of Quantum Mechanics", Ch. 14.4 (Spin dynamics), and in great detail in Claude Cohen-Tannoudji et al., "Quantum Mechanics" Vol 1., Ch.4, complement F_{IV}, (Spin 1/2 particle in a static magnetic field and a rotating field: magnetic resonance). See, even the title of that section of the text makes you want to read it and work through it!

19. What is the difference between T_2 and T_2^*?

20. What is the difference between T_1 and T_2?

21. Can our apparatus measure the spins of other nuclei besides H? I mean, don't other nuclei have protons in them? Further, resonance simply a matter of matching energy gaps to the appropriate photon? If one could trap a vial of Neon gas for example, could one in principle see a signal with the present apparatus?

22. Describe in detail how one would use the apparatus (set up the electronics) to create a signal as shown in figure 3.

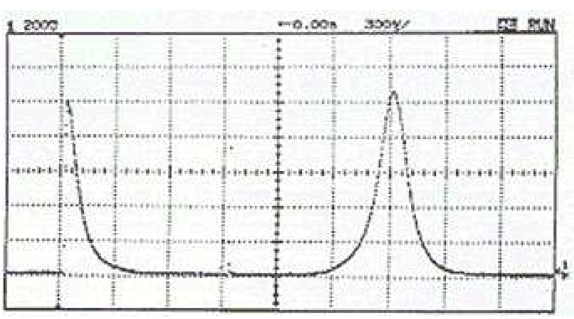

FIG. 3. In this figure we see a 'spin-echo' signal. How can one achieve such a signal with the apparatus, and what does the signal mean? What can one measure with it?

23. What role does the presence and peculiar structure of chemical bonds play in the measurements of the fundamental parameters? What sort of compounds can be analyzed with the present apparatus?

24. Do a literature search to find reported values of the fundamental parameters. How do the measured values of the fundamental parameters compare with values found in the literature?

III. RABI OSCILLATIONS

1. What is the experiment designed to measure? What are the principal results (giving figures of merit, with units, where possible)?

2. How is the experiment designed to measure those quantities?

3. Sketch a block diagram of the apparatus and label all the principal parts. What does each part do?

4. What difficulties are encountered typically (physical, technological, and so on), and how does the design permit these things to be overcome?

5. Read section 4-D (in the Optical Pumping TeachSpin manual) carefully and explain in detail how to find the resonance, that is, how to set up the experiment to acquire the data shown in Fig 4 below.

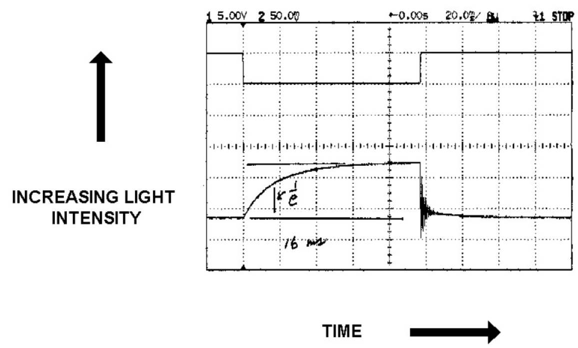

FIG. 4. Time dependence of the transmitted light intensity vs. RF amplitude (Figure 4D-1)

6. Referring to the zero field resonance (familiar from the Optical Pumping experiment) as ZFG, why is the magnetic field at which the low field resonance occurs for ^{87}Rb closer to the ZFG than that of ^{85}Rb? What determines the effective magnetic moment of the rubidium atoms in the gas phase in this experiment?

7. What are Rabi oscillations? In this experiment, why do they decay? Do be sure to review the relevant section of F. D. Colegrove, L. D. Schearer, and G. K. Walters, *Polarization of ^3He gas by Optical Pumping,* Phys. Rev. **132**, 2561 (1963).

8. Is this measurement different than NMR measurements? If so, how? Explain how to do an NMR measurement *that would directly measure the nuclear magnetic moment of the Rb isotopes* and how this differs from the measurements outlined in section 4-D in the manual. Before seeking to answer this question, write down your guess for which isotope has the bigger *nuclear magnetic moment.* For questions like this it is better to cheat than to guess. I take that back. Guess, so you have a thought in mind, then 'cheat', i.e., find a research paper that is especially pertinent and try to learn from them. Let me suggest three: 1)S. MILLMAN AND M. FOX, "Nuclear Spins and Magnetic Moments of ^{87}Rb and ^{85}Rb," Phys. Rev. **50** 220 (1936), and 2) E. Yasaitis and B. Smaller, "Nuclear Magnetic Moment of Rb^{85}, Rb^{87}, and I^{127}, Phys. Rev.

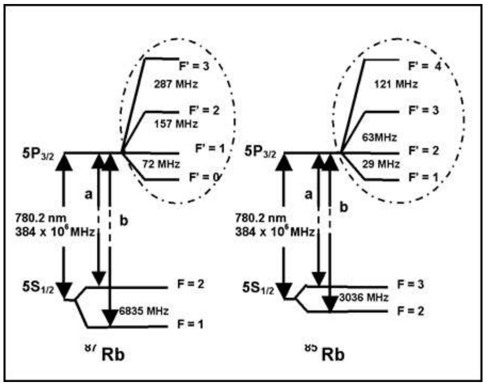

FIG. 5. Partial energy level diagram for Rb I, showing the split ground state $^2S_{1/2}$ and an excited state, $^2P_{3/2}$, for the two naturally occurring isotopes of Rb I.

82, 750 (1951), and of course, J.E. Nafe and E.B. Nelson, "The Hyperfine Structure of Hydrogen and Deuterium, " Phys. Rev. **73** 718 (1948).

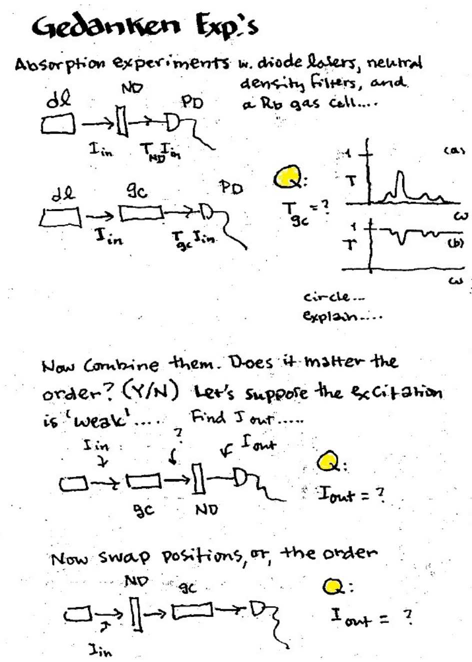

FIG. 6. Find the answers to the 3 questions depicted above.

IV. HIGH RESOLUTION LASER SPECTROSCOPY, AKA HYPERFINE STRUCTURE OF THE GROUND STATE OF RB I (AT USD ANYWAY)

1. What is the experiment designed to measure? What are the principal results (giving figures of merit, with units, where possible)?

2. How is the experiment designed to measure those quantities?

3. Sketch a block diagram of the apparatus and label all the principal parts. What does each part do?

4. What difficulties are encountered typically (physical, technological, and so on), and how does the design permit these things to be overcome?

5. What spectral resolution is required to 'resolve' electric dipole transitions to the ground state from these two states. Are such transitions allowed? Are electric dipole transitions allowed *between* these states? If not, are there any?

6. Have a look at the partial energy level diagram for the ground state of Rb I, fig 5. Draw for yourself an *absorption* spectrum commensurate with it that is both quantitatively and qualitatively correct. Take a shot, make some order of magnitude calculations, and commit yourself to an informed guess–make a sketch, label your axes, and be ready to defend your choices. How many lines (dips) do you expect to see, taking into consideration both isotopes?

7. How does the random thermal motion of the atoms in the gas cell affect the absorption spectrum? Why, without taking really drastic measures, would one see (if one had a really narrow line width laser, with a line width, say, or 1 MHz), just 4 blobby dips, and, since 'blobby dips', is a precise technical term, estimate what the width of the dips would be at half their depth.

8. What is saturation-absorption spectroscopy? How does it work? And by 'work' I mean, precisely how are the separate transitions to the separate hyperfine states, in the excited state, from the separate hyperfine states in the ground state resolved? Use a 'Velocity space diagram' (VSD) for the atoms in the ground state, an energy level diagram (Partial Grotrian diagram, or ELD), and the Doppler shift to depict and explain SubDoppler Laser spectroscopy.

9. It is suggested in the readings that 'crossover resonances' occur. What are they, and what causes them? Use a VSD for the atoms in the ground state, an energy level diagram (ELD), and the Doppler shift to depict and explain crossover resonances.

10. We'll use a lock-in amplifier to measure the hyperfine structure of the excited states of the rubidium isotopes.

11. There are 3 questions deployed in the thought experiment meant to illustrate saturated absorption, found in the figure below (figure 6). Answer these.

12. What is the physical basis of the measurement of absorption, using the lock-in detection? How do lock-in amplifiers work?

13. Taking the definition of resolving power R, to be the dimensionless ratio, $R = \lambda/\Delta\lambda$, what is the resolving power of the set up without crossing the probe and pump beams, and, without phase-sensitive detection? What is the resolving power, using the technique of saturation-absorption spectroscopy, and with phase-sensitive detection?

14. What is the signal to noise ratio (speaking of 'resolving' the hyperfine structure of the transition)? Try to estimate the extent to which the lock-in amplifier improves the signal to noise ratio. Express your result in $dB's$ (you will want to look up decibels).

15. What is phase-sensitive detection, and what is a lock-in amplifier?

16. What is an etalon, and how does it work? About how long should a cavity be if it is to have a 'free-spectral-range' of 300 MHz?

17. Where in the study of Quantum Mechanics does 'hyperfine structure' arise? What aspect of quantum theory is the student trying to understand?

18. Can one discover, *measure* quantum numbers with the technique of saturation absorption spectroscopy as applied in this experiment? If so, which? What does one discover about Rb and its isotopes?

19. If we could do this same experiment for hydrogen, what would we see? Is our technique, and the lasers we have laying around sufficient to acquire the same sort of signals? Speculate.

V. PLASMA PHYSICS OF ION ACOUSTIC WAVES

1. What is the experiment designed to measure? What are the principal results (giving figures of merit, with units, where possible)?

2. How is the experiment designed to measure those quantities?

3. Sketch a block diagram of the apparatus and label all the principal parts. What does each part do?

4. What difficulties are encountered typically (physical, technological, and so on), and how does the design permit these things to be overcome?

5. Describe the plasma state of matter. Is any collection of confined charged particles a plasma? Is a metal a plasma? What about a burning match or candle? What about the exhaust of a Saturn V rocket? Where in that engine is the 'gas' most likely to be in the plasma state? Why? What if one could heat up normal air to a temperature of 3.5kiloKelvins?

6. For the purpose of creating and experimentally examining a magnetically confined laboratory plasma, calculate (for the electron plasma density, temperature, and neutral pressure of, $n_e = 1 \times 10^9 cm^{-3}$, $T_e = 1eV$, $P_o = 0.5mTorr$, respectively)
   (a) the Debye length,
   (b) the average number of ions in a Debye sphere,
   (c) the quantity $\omega_p \tau$, the product of the so-called plasma frequency and the collision time (between electrons and neutrals). Are these quantities what they should be in order to insure that collective effects characterize our ionized gas?
   (d) What are some of the other collective effects possible in a plasma? Give one example and explain.

7. What is a Langmuir Probe? What does it measure?
   (a) How does the ratio of the electron and ion saturation currents compare with your expectation? How well can you measure the ratio?
   (b) Does the probe current-voltage characteristic indicate the presence of the so called primary electrons?
   (c) Using 'simple' probe theory, what is the voltage difference between the *floating* and *plasma potentials* in a plasma of density $1 \times 10^{11} cm^{-3}$. If $kT_e = 2eV$?

8. Why do we need a vacuum chamber for this experiment? Why do electron microscopes, Scanning tunneling microscopes, particle accelerators, and so on, need vacuum chambers? Why are semiconductor wafers and magnetic hard drives prepared in ultra high vacuum chambers (and in clean rooms)?

9. If a large grid or electrode is immersed in the plasma and biased positively (with respect to the plasma space potential), it is found that the electron temperature *rises*. Why is that? By the way, such a grid is called a Maxwell Demon (the plasma version).

10. How do the temperature and density of the plasma electrons depend on the discharge parameters?

11. read the file, "How to derive the dispersion relation for sound (acoustic) waves", found on our public course website (plasma readings). Complete and discuss solutions to tutorial questions 1 and 2.

12. What is the physical basis of the phenomena (IAWs, ion acoustic waves)? Further, how well do theory and experiment agree? Conceive of calculations done with collected data that most directly compare theory and experiment. Use your judgment. Are their aspects of the experiment that are

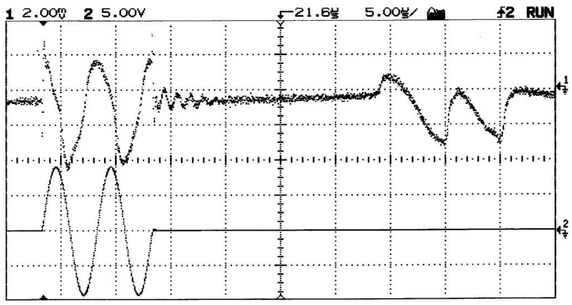

FIG. 7. Grid and probe are separated by some variable distance; the time delay between the tone burst applied to the grid and the moment of its appearance on the probe can be varied by changing this distance.

not treated by theory (the theory you are using to interpret the data) that could complicate the application of that theory?

13. Just how far does the analogy between sound waves in neutral gas and 'ion acoustic waves' extend? Are there effects found in one and not the other?

14. Why is it a good idea to chose excitation frequencies such that $f_{in} < f < f_{pi}$, where f_{in} is the ion-neutral collision frequency, and f_{pi} is the ion plasma frequency? Roughly, the product of the mean free path and the ion-neutral collision frequency is the ion thermal speed. One can make an estime of the ion neutral collision frequency then from two approximate facts for plasma discharges of the kind we produce in the multidipole chamber, 1) the ion temperature is roughly room temperature or thereabout, and 2) the mean free path for ion-neutral collision is around 5 cm, of thereabout.

15. If the distance between the grid and the probe is 3cm (for the data depicted in figure 7)

    (a) Estimate the group velocity and electron temperature. Describe the chain of inferences that permits this estimation. For the sake of simplicity, suppose that $k\lambda_D \ll 1$. This will be a crude estimate because one only has the single delay time.

    (b) Estimate the uncertainty in the group velocity, if one knows from other measurements that displacement uncertainty is 0.2cm; justify or explain your estimate. For some insight into phase vs. group velocity, confer with the 180E lab manual, Ch.IV (ion acoustic waves, see esp. figure IV-6d, except that since we have only 1 delay pulse, a defensible choice for delay time is measured from the center of voltage applied to the grid, to the center of propagating signal).

Equations of the Traveling Waves. The equations of the traveling electric wave polarized in the y direction and the traveling magnetic wave polarized in the z direction, each traveling in the positive x direction, may be written

$$E_y = E_{y_0} \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) \quad (3)$$

and

$$H_z = H_{z_0} \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) \quad (4)$$

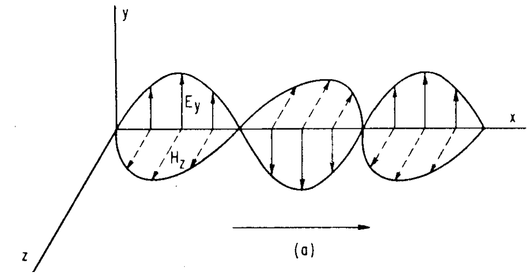

FIG. 8. Equations for the electric and magnetic fields and graphical representations of the same

VI. ELECTROMAGNETIC STANDING WAVES

It is expected that the student will be conversant with the article in the Physics Teacher (TPT), "Looking for radio waves with a simple radio wave detector", 2011, vol.49 page 2011, and the chapter on Hertz's discovery of traveling electromagnetic waves in the book by Shamos, "Famous experiments in Physics". From a physics laboratory manual popular in the early post-war era in the US, one finds snippets like those given in figure 8.

1. Show that the equations depicted in figure 8 lead to standing waves, say by simply adding waves traveling in opposite directions.

2. What is the wavelength associated with an ac source of 60 cycles per sec? With a DC source? What frequency corresponds to wave length 1mm?

3. Suppose for a particular set-up as described in the TPT paper, a standing wave is created by reflection, and that maxima and minima are found with the detectors, and that some distance from the metal sheet, there is a 'node'. If then one measures the distance to the next node, what is the wavelength of the waves? Introduce suitable notation. Explain.

4. If one discovers by measurement that for one of the fields, the metal sheet constitutes a node, it follows that there is an more or less exactly $\lambda/4$ from the boundary. Here is my problem: a wave heading for the boundary travels only $\lambda/4$ and then reflects, and then travels another distance of $\lambda/4$ *where it interferes with itself.* So the path length difference is half of a wavelength! But shouldn't that lead to *destructive interference?!* Please resolve this paradox.

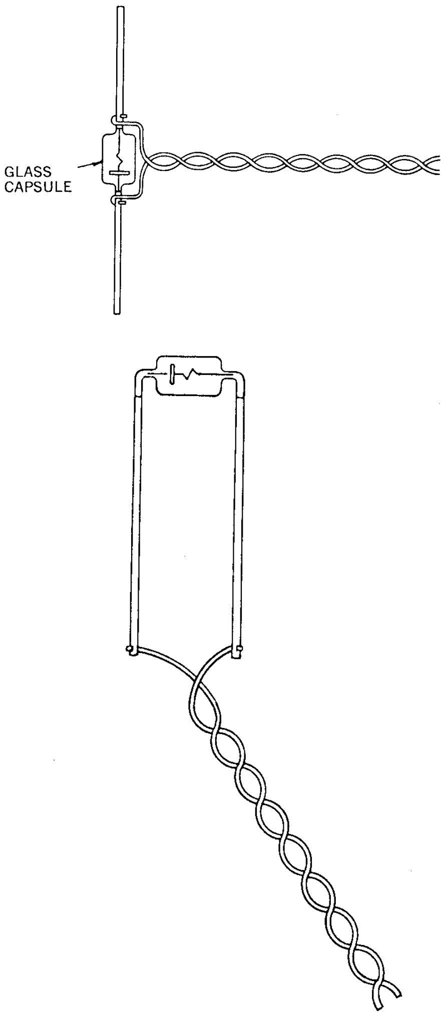

FIG. 9. detectors for the fields

5. Perform the following addition and interpret:

$$\begin{aligned} y(x,t) &= A\sin\left(kx-\omega t\right) \\ &+A\sin\left(kx-\omega t+2(L-x)k+\pi\right), \end{aligned} \tag{1}$$

simplifying as much as possible, where x is any point in space measured from the source, in between the source and the metal sheet, which is a distance L from the source, where k is one of the discrete values allowed for standing waves (which the student should be able to derive). Here y may be thought of as one of the traveling fields. Is the simplified expression that of a standing wave?

6. Without too much imagination, one can look at the pictures given in figure 9, and recognize two different types of field detectors, each dependent on a kind of fast response diode (in the TPT paper, it is the Schottky diode), but used in very different ways. Which figure is used to measure electric field magnitudes, and which is of principle use for magnetic fields? Which one responds to both (possibly). Explain