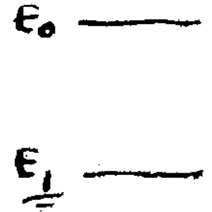
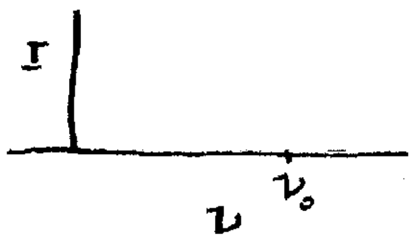
#1 **fine structure 480W, worksheet #2**

draw a very simple energy level diagram between a hypothetical ground state ($E_{\text{⏚}}$) and an excited state (E_0) which we will imagine is two fold degenerate.



(okay, I did it.... but draw in "the line" indicating the transition.....)



There is also an intensity vs frequency graph to capture the 'spectral line'. Take a minute to draw in the 'lines' on both diagrams. What physical process gives rise to a finite thickness to the 'spectral line' on the right? Jot down an answer.

#2

Imagine that the Hamiltonian, which gives the two fold degenerate energy eigenvalue E_0, may be represented as the matrix

$$\underline{\underline{H}}_0 = \begin{pmatrix} E_0 & 0 \\ 0 & E_0 \end{pmatrix},$$

----- see next page.........

and that there exists some 'small' perturbation, the matrix for which (in the basis in which the unperturbed Hamiltonian is diagonal) is shown below, and added to the unperturbed Hamiltonian, giving, to a first approximation, the complete Hamiltonian,

#4

Take the 'SPIN-ORBIT' perturbation Hamiltonian.

$$\hat{V}_{SO} = A\,\underbrace{\hat{L}}_{\propto \vec{B}}\cdot\underbrace{\hat{S}}_{\propto \vec{\mu}_e}$$

which arises by a relativistic transformation of the electric field experienced by an electron w. q#'s $n\ell\ell$ in the H atom (or any atom....)

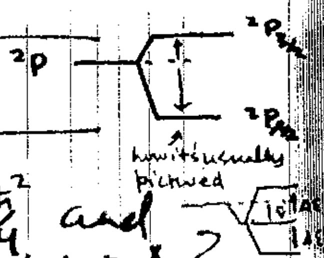
A depends on $\langle \frac{1}{r}\frac{dV}{dr}\rangle_{n\ell}$ ... and can be gotten experimentally for precise comparison w. theory...

the perturbation matrix

Find $\underline{\underline{V}}_{SO}$ in the ^2P ket basis and read off the new eigenvalues with the perturbation 'turned on'

$\delta\delta$ $\langle L'S'J'|LSJ\rangle = ?$

$\underline{\underline{V}}_{SO}$ =	$\lvert LS\,\tfrac{3}{2}\rangle$	$\lvert LS\,\tfrac{1}{2}\rangle$
$\langle LS\,\tfrac{3}{2}\rvert$		
$\langle LS\,\tfrac{1}{2}\rvert$		



Show that there is a level shift of $-\frac{A\hbar^2}{4}$ and that $\Delta E = \pm\frac{3A\hbar^2}{4}$. If the Gap is 4.6×10^{-5} eV, what is A?

also....is the value A consistent with the value of the 'fine structure constant?

$$\underline{\underline{H}} = \underline{\underline{H}}_0 + \underline{\underline{V}}', \qquad \begin{pmatrix} E_0 & 0 \\ 0 & E_0 \end{pmatrix} + \begin{pmatrix} V_{11}' & V_{12}' \\ V_{21}' & V_{22} \end{pmatrix} = \underline{\underline{H}} = \begin{pmatrix} \bar{V} + \Delta E & 0 \\ 0 & \bar{V} - \Delta E \end{pmatrix}$$

Try to find the new energy eigenvalues by diagonalizing the perturbation matrix (that is, by finding eigenvalues of the perturbation matrix). Find $\underline{\bar{V}}$, $\underline{\Delta E}$ in terms of the $\underline{\underline{V}}'$ matrix elements, E_0

#3

Now let's picture our results, and construe, or consider the applications. Go back to the 2 graphs, diagrams in part 1 and modify them in a way that make sense to you, but keep the 2 lines you drew .... use dashed lines for the new ones. or something. What about your new picture expresses the truth that $V' \ll H_0$.