Errata for "Univalence for inverse diagrams and homotopy canonicity"

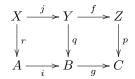
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1 A redundant axiom

I originally included the following property in the definition of a type-theoretic fibration category, but Joyal has pointed out that it follows from the other axioms. The following simple proof is due to the referee and Tamara von Glehn.

Lemma 1 (The referee; see Remark 3.8). Assuming conditions (1)–(5) for a type-theoretic fibration category, condition (6) also holds: given a commutative diagram:



if $g: B \to C$ and $gi: A \to C$ are fibrations, $i: A \to B$ is an acyclic cofibration, and both squares are pullbacks (hence $f: Y \to Z$ and $fj: X \to Z$ are fibrations), then $j: X \to Y$ is also an acyclic cofibration.

Proof. If p is a fibration, so is its pullback q, and thus pullback along q preserves acyclic cofibrations. Thus, factoring p according to (5), we may assume it is an acyclic cofibration. But then, since g and gi are fibrations, q and r are again acyclic cofibrations, and thus so is the composite ir = qj. Thus it suffices to prove the following lemma.

Lemma 2 (von Glehn). Assuming conditions (1)-(5), if gf and g are acyclic cofibrations, so is f.

Proof. Suppose given a commutative square as on the top below, where p is a

fibration; we must construct a lift.

Since $B \to 1$ is also a fibration and g is an acyclic cofibration, we have a lift $h: Z \to B$ such that hg = t. Now since gf is also an acyclic cofibration, we have a lift $k: Z \to A$ such that pk = h and k(gf) = s. Therefore, the composite kg has the properties that (kg)f = s and p(kg) = hg = t, as desired. (Note that this argument applies to the left class of any weak factorization system for which all maps to 1 lie in the right class.)

2 A small gap in Proposition 2.13

The definition of a type-theoretic model category implies that acyclic cofibrations are preserved by pullback along fibrations, which is also part of the definition of type-theoretic fibration category. However, the two notions of "acyclic cofibration" are not *a priori* the same! Both are defined by left lifting against fibrations, but in different categories: in the whole model category or in the subcategory of fibrant objects.

Thus, to complete the proof we need to show that if a map between fibrant objects in a type-theoretic model category has the left lifting property with respect to all fibrations between fibrant objects, then it is an acyclic cofibration. Fortunately, this is not difficult: if $f: A \to B$ is such a map, then factoring it as an acyclic cofibration followed by a fibration, f = pi, then p is a fibration between fibrant objects. Thus, by the retract argument, f is a retract of i, hence also an acyclic cofibration.