Those Quantifier Words

There are a handful of special words that are used in relation to quantifier strategies. These words are often hard to learn to use correctly, because they're (unfortunately) unsystematic and inconsistent. Here is a handy reference guide.

On the front we list for each quantifier strategy some of the different ways it can be introduced in English, with the special words in boldface. Then on the back we list, for each special word, all the strategies in which it can be used.

Prove forall

- **Let** x be an **arbitrary** real number.
- **Let** x be a real number.
- **Let** x be a real number **such that** $x > 0$.
- **Suppose** x is an **arbitrary** real number. *(and similar variations)*
- **Assume** x is an **arbitrary** real number. *(and similar variations)*
- **Fix** an **arbitrary** real number x. *(and similar variations)*

Use forall

- **Since** every square number is composite, **in particular** 16 is composite.
- **Since** every square number is composite, 16 is composite.
- **Since** $f(x) = g(x)$ for all x, **we have** $f(3) = g(3)$.

Prove exists

- **Let** $x = 1$; we will show $x^2 + 1 = 2$.
- **Choose** $x = 1$; we will show $x^2 + 1 = 2$.
- **Define** $x = 1$; we will show $x^2 + 1 = 2$.
- **Set** $x = 1$; we will show $x^2 + 1 = 2$.

Use exists

- **Let** p be an odd prime number.
- **Let** x be a real number **such that** $x^2 + x = 20$.
- **Choose** any odd prime number p. *(And similar variations)*
- **Fix** some odd prime number p. *(And similar variations)*
- **We may suppose** given some odd prime number p. *(And similar variations)*
- **Since** n is odd, **we have** an integer m **such that** $m = 2n$.

Arbitrary

- `prove forall`: Let x be an *arbitrary* real number.

Choose

- `prove exists`: *Choose* $x = 1$; we will show $x^2 + 1 = 2$. *(The author makes a specific choice)*
- `use exists`: *Choose* any odd prime number p. *(The reader is free to make any choice)*

Define / set

- `prove exists`: *Define* $x = 1$; we will show $x^2 + 1 = 2$.
- `prove exists`: *Set* $x = 1$; we will show $x^2 + 1 = 2$.
- (Also when giving a name to something to be studied further.)

In particular

- `use forall`: Since every square number is composite, *in particular* 16 is composite.

Fix

- `prove forall`: *Fix* an arbitrary real number x.
- `use exists`: *Fix* some odd prime number p.

Let

- `prove forall`: *Let* x be an arbitrary real number.
- `prove exists`: *Let* $x = 1$; we will show $x^2 + 1 = 2$.
- `use exists`: *Let* p be an odd prime number.
- (Also when giving a name to something to be studied further.)

Since

- `use forall`: *Since* every square number is composite, 16 is composite.
- (Also `use conditional`: *Since* P implies Q, and P, we have Q.)
- (Also when giving the reason for absolutely anything.)

Such that

- `prove forall`: Let x be a real number *such that* $x > 0$.
- `use exists`: Since n is odd, we have an integer m *such that* $m = 2n$.

Suppose / Assume

- `prove forall`: *Suppose* x is an arbitrary real number.
- `prove forall`: *Assume* x is an arbitrary real number.
- `use exists`: *We may suppose* given some odd prime number p.
- `use exists`: *We may assume* given some odd prime number p.
- (Also `prove conditional`: *Assume* $x^3 = 1$; we will show $x^5 = x^2$.)

We have

- `use forall`: Since $f(x) = g(x)$ for all x, *we have* $f(3) = g(3)$.
- `use exists`: Since n is odd, *we have* an integer m such that $m = 2n$.