Physics 140

HOMEWORK Chapter 4A

Q1. Figure 4-21 shows the path taken by a skunk foraging for trash food, from initial point $i$. The skunk took the same time $T$ to go from each labeled point to the next along its path. Rank points $a$, $b$, and $c$ according to the magnitude of the average velocity of the skunk to reach them from initial point $i$, greatest first.

$c = a > b$. Orient axes so that $i$ is to right, and $j$ is toward the top of the page. Going from $i$ to $a$, the average velocity was $-1/T \hat{i}$. Magnitude $= 1/T$.
Going from $i$ to $b$, the average velocity was $+1/(2T) \hat{i}$. Magnitude $= 1/2T$.
Going from $i$ to $c$, the average velocity was $+3/(3T) \hat{i}$. Magnitude $= 1/T$.

Q4. You are to launch a rocket, from just above the ground, with one of the following initial velocity vectors: (1) $\vec{v}_0 = 20\hat{i} + 70\hat{j}$, (2) $\vec{v}_0 = -20\hat{i} + 70\hat{j}$, (3) $\vec{v}_0 = 20\hat{i} - 70\hat{j}$, and (4) $\vec{v}_0 = -20\hat{i} - 70\hat{j}$. In your coordinate system, $x$ runs along level ground and $y$ increases upward. (a) Rank the vectors according to the launch speed of the projectile, greatest first. (b) Rank the vectors according to the time of flight of the projectile, greatest first.

(a) $(1) = (2) > (3) = (4)$. Or maybe all equal. The first two have equal upward components of velocity. The third and fourth are launched downward from just above the ground. Possibly they actually reach the launch speed before hitting the ground, which accounts for the “maybe all equal” above. Whether the $x$-component is positive or negative only affects whether they go to the right or to the left.
(b) $(1) = (2) > (3) = (4)$. Going upward produces a longer time of flight.

Q5. Figure 4-23 shows three situations in which identical projectiles are launched (at the same level) at identical initial speeds and angles. The projectiles do not land on the same terrain, however. Rank the situations according to the final speeds of the projectiles just before they land, greatest first.

$(a) > (b) > (c)$. The horizontal component is equal in each case. The vertical component is equal to its initial value for $(b)$. For $(a)$, the vertical component is greater than its initial because $v_{yf}^2 = v_{y0}^2 + 2a\Delta y$. Since the acceleration $a$ and $\Delta y$ are both negative, $2a\Delta y$ is positive, and $v_{yf} > v_{y0}$.

Q9. Figure 4-26 shows three paths for a football kicked from ground level. Ignoring the effects of air, rank the paths according to (a) time of flight, (b) initial vertical velocity component, (c) initial horizontal velocity component, and (d) initial speed, greatest first.

(a) all equal. In each case, the time-of-flight is twice the time an object would take to drop from the dashed line level.
(b) all equal. They all reach the same maximum height.
(c) $3 > 2 > 1$. The $v_x$ is a constant, and the times are equal, so the one which goes the greatest distance $\Delta x$ has the highest $v_x$.
(d) $3 > 2 > 1$. Speed $v = \sqrt{v_{0x}^2 + v_{0y}^2}$. Since the $v_{0y}$ is the same for all, the speed $v$ will rank the same as $v_{0x}$.

P6. An electron’s position is given by $\vec{r} = 3.00t\hat{i} - 4.00t^2\hat{j} + 2.00\hat{k}$, with $t$ in seconds and $\vec{r}$ in meters. (a) In unit-vector notation, what is the electron’s velocity $\vec{v}(t)$? At $t = 2.00$ s, what is (b) in unit-vector notation and as (c) a magnitude and (d) an angle relative to the positive direction of the $x$ axis?

(a) $\vec{v} = d\vec{r}/dt = 3\hat{i} - 8t\hat{j}$. The $z$-component is zero.
(b) $\vec{v}(2\text{ s}) = 3\hat{i} - 16\hat{j}$, where the units are m/s.
(c) $|\vec{v}(2\text{ s})| = \sqrt{3^2 + 16^2} = 16.3$ m/s.
(d) By inspecting a sketch of this vector, it is in quadrant IV. The magnitude of the angle is $|\theta| = \tan^{-1}(16/3) = 79.4^\circ$. This may be specified as 79.4$^\circ$ below the $+x$-axis, or as $-79.4^\circ$.

**P12.** At one instant a bicyclist is 40.0 m due east of a park’s flagpole, going due south with a speed of 10.0 m/s. Then 30.0 s later, the cyclist is 40.0 m due north of the flagpole, going due east with a speed of 10.0 m/s. For the cyclist in this 30.0-s interval, what are the (a) magnitude and (b) direction of the displacement, the (c) magnitude and (d) direction of the average velocity, and the (e) magnitude and (f) direction of the average acceleration?

We are given $\Delta \vec{r} = \vec{r}_2 - \vec{r}_1 = (+40 \hat{j} \text{ m}) - (+40 \hat{i} \text{ m}) = (-40 \hat{i} + 40 \hat{j}) \text{ m}$. We need to convert to magnitude and angle.

(a) magnitude: $|\Delta \vec{r}| = \sqrt{40^2 + 40^2} = 56.6 \text{ m}$.

(b) angle is in Quadrant II, so is 45$^\circ$ north of west, or 135$^\circ$.

Average velocity $\vec{v}_{\text{avg}} = \Delta \vec{r}/\Delta t$, so

(c) $|\vec{v}_{\text{avg}}| = |\Delta \vec{r}|/\Delta t = (56.6 \text{ m})/(30.0 \text{ s}) = 1.89 \text{ m/s}$.

(d) angle is same as part (b): $\vec{v}_{\text{avg}}$ is always in the same direction as $\Delta \vec{r}$.

Average acceleration $\vec{a}_{\text{avg}} = \Delta \vec{v}/\Delta t = [10 \hat{i} \text{ m/s} - (-10 \hat{j} \text{ m/s})]/(30.0 \text{ s}) = (0.333 \hat{i} + 0.333 \hat{j}) \text{ m/s}^2$, so

(e) $|\vec{a}_{\text{avg}}| = \sqrt{0.333^2 + 0.333^2} \text{ m/s}^2 = 0.471 \text{ m/s}^2$.

(f) angle is in Quadrant I, so is 45$^\circ$ north of east.

**P18.** A moderate wind accelerates a pebble over a horizontal $xy$ plane with a constant acceleration $\vec{a} = (5.00 \hat{i} + 7.00 \hat{j}) \text{ m/s}^2$. At time $t = 0$, the velocity is $(4.00 \text{ m/s}) \hat{i}$. What are the (a) magnitude and (b) angle of its velocity when it has been displaced by 12.0 m parallel to the $x$ axis?

We are given $\vec{v}_0$ and $\vec{a}$, and need $\vec{v}_f = \vec{v}_0 + \vec{a}t$. This will be easy, once we get the elapsed time $t$. We get $t$ from the $x$ motion. We know that $\Delta x = 12 \text{ m}$, and also that $\Delta x = v_{0x}t + (1/2)a_xt^2$, or, suppressing units:

$$2.5t^2 + 4t - 12 = 0.$$ Solve the quadratic to get $t = 1.532 \text{ s}$.

Plug to get $\vec{v}_f = [(4 + 5 \times 1.532) \hat{i} + (7 \times 1.532) \hat{j}] = (11.66 \hat{i} + 10.72 \hat{j}) \text{ m/s}$.

(a) Pythagoras: $|\vec{v}_f| = \sqrt{11.66^2 + 10.72^2} = 15.8 \text{ m/s}$.

(b) The angle, in Quadrant I, is $\theta = \tan^{-1}(10.72/11.66) = 42.6^\circ$.

**P27.** A certain airplane has a speed of 290.0 km/hr and is diving at an angle of 30.0$^\circ$ below the horizontal when the pilot releases a radar decoy (Fig. 4-33). The horizontal distance between the release point and the point where the decoy strikes the ground is $d = 700 \text{ m}$. (a) How long is the decoy in the air? (b) How high was the release point?

Recognize that “released” means that the decoy’s initial velocity is the same as the plane’s. We will need to work in meters and m/s, so 290 km/hr = 80.56 m/s. Taking $\hat{i}$ as horizontal and $\hat{j}$ as upward, $\vec{v}_0 = (69.77 \hat{i} - 40.28 \hat{j}) \text{ m/s}$.

(a) We have that $\Delta x = 700 \text{ m}$, and that $\Delta x = v_{0x}t$ (realizing that $a_x = 0$).

$t = (700 \text{ m})/(69.77 \text{ m/s}) = 10.03 \text{ s}$.

(b) $\Delta y = v_{0y}t - (1/2)gt^2 = (-40.28)(10.03) - (4.9)(10.03)^2 = -897 \text{ m}$.

The height of release is $-\Delta y = 897 \text{ m}$.

**P32.** You throw a ball toward a wall at speed 25.0 m/s and at angle 40.0$^\circ$ above the horizontal (Fig. 4-35). The wall is distance $d = 22.0 \text{ m}$ from the release point of the ball. (a) How far above the release point does the ball hit the wall? What are the (b) horizontal and (c) vertical components of its velocity as it hits the wall? (d) When it hits, has it passed the highest point on its trajectory?

Recognize a projectile motion problem.

The idea is to figure out when the ball hits the wall ($t_h$) and use that to answer the questions. Note that
\[ v_{0x} = + (25 \text{ m/s}) \cos 40^\circ = 19.15 \text{ m/s} \quad \text{and} \quad v_{0y} = + (25 \text{ m/s}) \sin 40^\circ = 16.07 \text{ m/s}. \]
\[ \Delta x = 22 \text{ m} = v_{0x} t_h \quad \Rightarrow \quad t_h = (22 \text{ m})/19.15 \text{ m/s} = 1.149 \text{ s} \]
(a) \[ \Delta y = v_{0y} t_h - (1/2)g t_h^2 = (16.07 \text{ m/s})(1.149 \text{ s}) - (1/2)(9.8 \text{ m/s}^2)(1.149 \text{ s})^2 = 11.99 \text{ m} = 12.0 \text{ m}. \]
(b) \[ v_x = v_{0x} = 19.15 \text{ m/s} \]
(c) \[ v_y = v_{0y} - gt_h = 16.07 \text{ m/s} - (9.8 \text{ m/s}^2)(1.149 \text{ s}) = +4.81 \text{ m/s}. \]
(d) Since \( v_y(t_h) > 0 \), the ball is still rising when it hits the wall.

P33. A plane, diving with constant speed at an angle of 53.0° with the vertical, releases a projectile at an altitude of 730 m. The projectile hits the ground 5.00 s after release. (a) What is the speed of the plane? (b) How far does the projectile travel horizontally during its flight? What are the (c) horizontal and (d) vertical components of its velocity just before striking the ground?

Recognize a projectile motion problem. We don’t know \( v_0 \), but we do know its angle. Since 53° is the angle with the vertical, the plane’s angle with the horizontal is 37°. If you missed this point, it illustrates the value of sketching the situation. So: \( v_{0x} = v_0 \cos 37° \) and \( v_{0y} = -v_0 \sin 37° \). We are given \( \Delta y = -730 \text{ m} \) and \( t_h = 5 \text{ s} \). This suggests we should try using the equation for \( \Delta y \):
(a) \[ \Delta y = v_{0y} t_h - (1/2)g t_h^2 \]
\[ v_{0y} = [\Delta y + (1/2)g t_h^2]/t_h = [-730 + (1/2)(9.8)(5^2)]/5 = -121.5 \text{ m/s} \]
\[ v_0 = -v_{0y}/\sin 37° = (-121.5)/0.602 = 202 \text{ m/s}. \]
(b) \[ v_{0x} = (202 \text{ m/s}) \cos 37° = 161.3 \text{ m/s}, \]
\[ \Delta x = v_{0x} t_h = (161.3)(5) = 807 \text{ m}. \]
(c) \[ v_x = v_{0x} = 161 \text{ m/s}. \]
(d) \[ v_y = v_{0y} - gt_h = (-121.5) - (9.8)(5) = -171 \text{ m/s}. \]

P35. A rifle that shoots bullets at 460 m/s is to be aimed at a target 45.7 m away. If the center of the target is level with the rifle, how high above the target must the rifle barrel be pointed so that the bullet hits dead center?

Since the impact point is at the same level as the launch point, we can use the range equation, Eq 4.26. We expect to solve this for the angle, then use trig to answer the question.
\[ R = v_0^2 \sin 2\theta_0/g \quad \Rightarrow \quad \sin 2\theta_0 = gR/v_0^2 = (9.8)(45.7)/(460^2) = 0.002117. \]
The sine of an angle should be dimensionless. You should check that this is the case.
\[ \Rightarrow \quad 2\theta_0 = 0.1213^\circ \quad \Rightarrow \quad \theta_0 = 0.06063^\circ. \]
The height above the bullseye \( h \) is given by
\[ \tan \theta_0 = h/R \quad \Rightarrow \quad h = R \tan \theta_0 = (45.7)(0.001058) = 0.0484 \text{ m} = 4.84 \text{ cm}. \]

P39. In Fig. 4-37, a ball is thrown leftward from the left edge of the roof, at height \( h \) above the ground. The ball hits the ground 1.50 s later, at distance \( d = 25.0 \text{ m} \) from the building and at angle \( \theta = 60.0^\circ \) with the horizontal. (a) Find \( h \). (Hint: One way is to reverse the motion, as if on video.) What are the (b) magnitude and (c) angle relative to the horizontal of the velocity at which the ball is thrown? (d) Is the angle above or below the horizontal?

Recognize a projectile motion problem. Make \(+x\) leftward to correspond to the diagram; make \(+y\) upward. Write the proj. motion equations substituting for what is known. Use \( \phi \) for the launch angle, since the impact angle has already been designated \( \theta \). The angle \( \phi \) may turn out to be positive or negative. Suppress units.
\[ \Delta x = 25 = (v_0 \cos \phi)(1.5) \]
\[ \Delta y = -h = (v_0 \sin \phi)(1.5) - (4.9)(1.5^2) \]
\[ v_x = v_0 \cos \phi \]
\[ v_y = v_0 \sin \phi - (9.8)(t) \]
A key point will be that
\[ v_y(1.5\ s)/v_x(1.5\ s) = \tan 60^\circ. \]

Let's start with the \( \Delta x \) equation. Rearrange to:

\[ v_0 \cos \phi = \frac{25}{1.5} = 16.67\ \text{m/s} \quad \text{Eq 1} \]

From the fact that the impact angle is at \( \theta = 60^\circ \), we have that

\[ v_y(1.5\ s) = -(16.67\ \text{m/s})(\tan 60^\circ) = -28.87\ \text{m/s}. \quad \text{Eq 2} \]

From Eq 2 and the \( v_y \) equation,

\[ -28.87 = v_0 \sin \phi - (9.8)(1.5) \quad \Rightarrow \]

\[ v_0 \sin \phi = -14.17\ \text{m/s}. \quad \text{Eq 3} \]

(a) At this point, plug the \( \Delta y \) equation:

\[ -h = -14.17(1.5) - (1/2)(9.8)(1.5^2) = -32.3\ \text{m} \quad \Rightarrow \]

\[ h = 32.3\ \text{m}. \]

(b) Use Pythagoras on Eqs 1 and 3 to get

\[ v_0 = \sqrt{(-14.17)^2 + 16.67^2} = 21.9\ \text{m/s}. \]

(c) Divide Eq 3 by Eq 1 to get:

\[ \tan \phi = -14.17/16.67 \quad \Rightarrow \quad \phi = -40.4^\circ \]

(d) Since \( \phi \) is negative, the ball was thrown from below horizontal.