Q7. A rhinoceros beetle rides the rim of a horizontal disk rotating counterclockwise like a merry-go-round. If the beetle then walks along the rim in the direction of the rotation, will the magnitudes of the following quantities (each measured about the rotation axis) increase, decrease, or remain the same (the disk is still rotating in the counterclockwise direction): (a) the angular momentum of the beetle-disk system, (b) the angular momentum and angular velocity of the beetle, and (c) the angular momentum and angular velocity of the disk? (d) What are your answers if the beetle walks in the direction opposite the rotation?

(a) The total angular momentum stays constant.
(b) increase. The beetle is moving faster.
(c) decrease. Total $L$ remains constant, so the disk has to slow.
(d) System $L$ stays constant; $L_{\text{beetle}}$ decreases; $L_{\text{disk}}$ increases.

Q8. Figure 11-27 shows an overhead view of a rectangular slab that can spin like a merry-go-round about its center at $O$. Also shown are seven paths along which wads of bubble gum can be thrown (all with the same speed and mass) to stick onto the stationary slab. (a) Rank the paths according to the angular speed that the slab (and gum) will have after the gum sticks, greatest first. (b) For which paths will the angular momentum of the slab (and gum) about $O$ be negative from the view of Fig. 11-27?

We will assume that the mass of the gum is small compared to the mass of the slab. Else, the matter gets complicated. The final $\omega$ will be approximately proportional to the angular momentum supplied by the gum. This $L$ is simply $L = m_{\text{gum}}v_{\text{gum}}b$, where $b$ is the “impact parameter,” the perpendicular distance from $O$ to the extended path of the gum.

(a) $|L_4| > |L_6| > |L_7| > |L_1| > |L_2| = |L_3| = |L_5| = 0$.
(b) Rotation will be CW (negative) for paths 1, 4, and 7.

Q10. Figure 11-29 shows a particle moving at constant velocity and five points with their $xy$ coordinates. Rank the points according to the magnitude of the angular momentum of the particle measured about them, greatest first.

Again, the magnitude of $L$ is $L = mv\mathbf{b}$, where only $b$ varies.
$|L_6| > |L_4| > |L_3| > |L_2| = |L_1| = 0$.

P35. At time $t$, the vector $\vec{r} = 4.0t^2\hat{i} - (2.0t + 6.0t^2)\hat{j}$ gives the position of a 3.0 kg particle relative to the origin of an $xy$ coordinate system ($\vec{r}$ is in meters and $t$ is in seconds). (a) Find an expression for the torque acting on the particle relative to the origin. (b) Is the magnitude of the particle’s angular momentum relative to the origin increasing, decreasing, or unchanged?

This is an exercise in cross products in the $\hat{i}$-$\hat{j}$-$\hat{k}$ notation. Given $\vec{r}$, we compute $\vec{v} = d\vec{r}/dt$, then $\vec{L} = \vec{r} \times (m\vec{v})$, and finally $\vec{\tau} = d\vec{L}/dt$. All units come out in SI, with some possible confusion in that the constants carry implied units; e.g., the “4.0” is 4.0 m/s$^2$.

$\vec{r} = 4.0t^2\hat{i} - (2.0t + 6.0t^2)\hat{j}$
$\vec{v} = d\vec{r}/dt = 8.0t\hat{i} - (2.0 + 12.0t)\hat{j}$
$\vec{L} = (3 \text{ kg})(r_xv_y - r_yv_x)\hat{k} = (3 \text{ kg})[(4t^2)(-1)(2 + 12t) - (t + 12)(2t + 6t^2)(8t)]\hat{k} = 24t^2 \text{ kg m}^2/\text{s}$.
$\vec{\tau} = d\vec{L}/dt = 48 \hat{k} \text{ N m}$.

(b) Since the signs of both $\vec{\tau}$ and $\vec{L}$ are the same (positive), $|\vec{L}|$ is increasing.
P39. The angular momentum of a flywheel having a rotational inertia of 0.140 kg m² about its central axis decreases from 3.00 to 0.800 kg m²/s in 1.50 s. (a) What is the magnitude of the average torque acting on the flywheel about its central axis during this period? (b) Assuming a constant angular acceleration, through what angle does the flywheel turn? (c) How much work is done on the wheel? (d) What is the average power of the flywheel?

(a) \( \tau_{\text{avg}} = \Delta L/\Delta t = (0.80 \text{ kg m}^2/\text{s} - 3.00 \text{ kg m}^2/\text{s})/(1.5 \text{ s}) = -1.47 \text{ N m}, \) or \(|\tau| = 1.47 \text{ N m} \).

(b) We need the angular velocities. Use \( L = I\omega \Rightarrow \omega = L/I \)

\[ \omega_0 = (3.0 \text{ kg m}^2/\text{s})/(0.14 \text{ kg m}^2) = 21.43 \text{ rad/s} \text{ and } \omega_f = 5.71 \text{ rad/s}. \]

The easiest way is to get \( \omega_{\text{avg}} = 13.57 \text{ rad/s} \) and

\[ \Delta \theta = \omega_{\text{avg}} t = 20.36 \text{ rad.} \]

(c) The easiest way is \( W = \tau_{\text{avg}}|D\theta| = (-1.47 \text{ N m})(20.36 \text{ rad}) = -29.9 \text{ J.} \)

(d) \( P_{\text{avg}} = (-29.9 \text{ J})/(1.5 \text{ s}) = 20.0 \text{ W.} \)

P43. In Fig. 11-47, two skaters, each of mass 50 kg, approach each other along parallel paths separated by 3.0 m. They have opposite velocities of 1.4 m/s each. One skater carries one end of a long pole of negligible mass, and the other skater grabs the other end as she passes. The skaters then rotate around the center of the pole. Assume that the friction between skates and ice is negligible. What are (a) the radius of the circle, (b) the angular speed of the skaters, and (c) the kinetic energy of the two-skater system? Next, the skaters pull along the pole until they are separated by 1.0 m. What then are (d) their angular speed and (e) the kinetic energy of the system? (f) What provided the energy for the increased kinetic energy?

(a) From symmetry, the radius will be half the separation distance: \( R = (3 \text{ m})/2 = 1.5 \text{ m.} \)

(b) They will continue at the same speed, but the tension in the rod will cause them to go in a circle. \( \omega = v/r = (1.4 \text{ m/s})/(1.5 \text{ m}) = 0.933 \text{ rad/s.} \)

(c) The easiest way to get \( K \) is via \((1/2)mv^2\), but let’s do it with \((1/2)I\omega^2\). We got \( \omega \) in part (b), but need \( I \) about the center of the pole:

\[ I = \Sigma m_i r_i^2 = (50 \text{ kg})(1.5 \text{ m})^2 + (50 \text{ kg})(1.5 \text{ m})^2 = 225 \text{ kg m}^2. \]

\[ K = (1/2)(225 \text{ kg m}^2)(0.933 \text{ rad/s})^2 = 98 \text{ J.} \]

(d) The radius is now 0.5 m. \( I_f = 2 \cdot (50 \text{ kg})(0.5 \text{ m})^2 = 25 \text{ kg m}^2. \) Since there is zero torque on the system, \( L = I\omega \) is conserved:

\[ \omega_f = I_f \omega_0/I_f = 8.40 \text{ rad/s.} \]

(e) \( K_f = (1/2)I_f\omega_f^2 = (1/2)(25 \text{ kg m}^2)(8.4 \text{ rad/s})^2 = 882 \text{ J.} \)

(f) The work the skaters did by pulling themselves inward.

P49. Two disks are mounted (like a merry-go-round) on low-friction bearings on the same axle and can be brought together so that they couple and rotate as one unit. The first disk, with rotational inertia 3.30 kg m² about its central axis, is set spinning counterclockwise at 450 rev/min. The second disk, with rotational inertia 6.60 kg m² about its central axis, is set spinning counterclockwise at 900 rev/min. They then couple together. (a) What is their angular speed after coupling? If instead the second disk is set spinning clockwise at 900 rev/min, what are their (b) angular speed and (c) direction of rotation after they couple together?

Recognize a conservation of angular momentum problem, with the system consisting of both disks. \( L_f = L_0 \). You could convert rev/min to rad/s, but in a simple problem where all angular velocities are in rev/min, we can use that unit and have angular momentum be in \( \text{ kg m}^2 \text{ rev/min}. \)

(a) \( L_0 = +(3.30 \text{ kg m}^2)(450 \text{ rev/min}) + (6.60 \text{ kg m}^2)(900 \text{ rev/min}) = 7425 \text{ kg m}^2 \text{ rev/min}. \)

\[ L_0 = L_f = (I_f)\omega_f \Rightarrow \omega_f = L_0/I_f = (7425 \text{ kg m}^2 \text{ rev/min})/(9.9 \text{ kg m}^2) = +750 \text{ rev/min}. \]

(b) \( L_0 = +(3.30 \text{ kg m}^2)(450 \text{ rev/min}) - (6.60 \text{ kg m}^2)(900 \text{ rev/min}) = -4455 \text{ kg m}^2 \text{ rev/min}. \)

\[ L_0 = L_f = (I_f)\omega_f \Rightarrow \omega_f = L_0/I_f = (-4455 \text{ kg m}^2 \text{ rev/min})/(9.9 \text{ kg m}^2) = -450 \text{ rev/min}. \]

(c) clockwise, as implied by the negative sign in part (b).
P63. In Fig. 11-56, a 30 kg child stands on the edge of a stationary merry-go-round of radius 2.0 m. The rotational inertia of the merry-go-round about its rotation axis is 150 kg m². The child catches a ball of mass 1.0 kg thrown by a friend. Just before the ball is caught, it has a horizontal velocity of magnitude 12 m/s, at angle $\phi = 37^\circ$ with a line tangent to the outer edge of the merry-go-round, as shown. What is the angular speed of the merry-go-round just after the ball is caught?

Fairly basic problem on conservation of angular momentum; doesn’t justify the 3-dot difficulty marking.

\[ L_f = L_0, \]
\[ L_f = I_f \omega_f, \]
\[ I_f = I_{MGR} + (31 \text{ kg})(2 \text{ m})^2 = 274 \text{ kg m}^2. \]
\[ L_f = L_0 = |\vec{r} \times \vec{p}| = (2 \text{ m})(12 \text{ kg m/s})(\sin 53^\circ) = 19.2 \text{ kg m}^2/\text{s}. \]
\[ \omega_f = L_0/I_f = (19.2 \text{ kg m}^2/\text{s})/(274 \text{ kg m}^2) = 0.0700 \text{ rad/s}. \]

P71. In Fig. 11-60, a constant horizontal force of magnitude 12 N is applied to a uniform solid cylinder by fishing line wrapped around the cylinder. The mass of the cylinder is 10 kg, its radius is 0.10 m, and the cylinder rolls smoothly on the horizontal surface. (a) What is the magnitude of the acceleration of the center of mass of the cylinder? (b) What is the magnitude of the angular acceleration of the cylinder about the center of mass? (c) In unit-vector notation, what is the frictional force acting on the cylinder?

Recognize Newton’s Second Law, with rotations.

(a) XFBD for cylinder: Draw a circle. $F_{\text{app}}$ at top, to right; $n$ at bottom, upward; $mg$ at center, downward; $f_s$ at bottom, leftward(?). $\vec{a}$ to right. $\alpha$ CW. Make $+x$ rightward, and take $\alpha$ as positive CW, in contrast to standard convention, so that we don’t deal with a negative $\alpha$. The ?? about $f_s$ is because we don’t know which way it is. In fact, it turns out to be to rightward.

Take torque about center of wheel. The forces $n$ and $mg$ exert zero torque.

Torque eq: $F_{\text{app}}R + f_sR = I\alpha$  
Rewrite, dividing through by $R$ and using $a = \alpha R$:

Alt form Torque eq: $F_{\text{app}} + f_s = Ia/R^2$.

\[ x\text{-eq: } F_{\text{app}} - f_s = ma. \]
Add to Alt form torque eq:

\[ 2F_{\text{app}} = (m + I/R^2)a \rightarrow a = 2F_{\text{app}}/(m + I/R^2) = \]
\[ = 2F_{\text{app}}/(m + m/2) = (4/3)F_{\text{app}}/m = (4/3)(12 \text{ N})/(10 \text{ kg}) = 1.6 \text{ m/s}^2. \]

Since $a > F_{\text{app}}/m$, it’s clear that $f_s$ is rightward.

Solve the $x$ eq for $f_s$: $f_s = F_{\text{app}} - ma = 12 \text{ N} - (10 \text{ kg})(1.6 \text{ m/s}^2) = -4.00 \text{ N}$. This comes out negative, confirming that it is rightward.

(b) $\alpha = a/R = (1.6 \text{ m/s}^2)/(0.1 \text{ m}) = 16 \text{ rad/s}^2$.

(c) $f_s = +4.00 \text{ N}$. The $\hat{i}$ points rightward, which is, again, the correct direction of $f_s$.

This brings up the question of what would happen if the surface were frictionless (or if $\mu_k$ and $\mu_s$ were reduced to the point that it slips. It would accelerate, but spin up even faster, like a drag racer leaving the gate.

Q for the student: since you get more acceleration that if it were frictionless, does that mean you actually gain more energy than the work you do?
**P93.** A body of radius $R$ and mass $m$ is rolling smoothly with speed $v$ on a horizontal surface. It then rolls up a hill to a maximum height $h$. (a) If $h = 3v^2/4g$, what is the body’s rotational inertia about the rotational axis through its center of mass? (b) What might the body be?

Recognize a conservation of energy problem with rolling without slipping.

(a) $U_f + K_f = U_0 + K + 0$. Here, $U_f = mgh = (3/4)mgv^2/g = (3/4)mv^2$. Again, $v$ is the speed at the bottom.

$U_0 = 0$ by choosing $y = 0$ at the bottom; $K_f = 0$ because it stops at its highest point. and

$K_0 = (1/2)mv^2 + (1/2)I\omega^2 = (1/2)mv^2 + (1/2)(I/R^2)v^2 = (1/2)(m + I/R^2)v^2$. Here, the “rolls without slipping” has been used to eliminate $\omega$ via $v = R\omega$.

$(1/2)(m + I/R^2)v^2 = (3/4)mv^2$. Solve for the expression $I/R^2$:

$$I/R^2 = m/2 \Rightarrow I = (1/2)mR^2$$

(b) Disk, or cylinder.

**P66 Opt** In Fig. 11-58, a small 50 g block slides down a frictionless surface through height $h = 20$ cm and then sticks to a uniform rod of mass 100 g and length 40 cm. The rod pivots about point $O$ through angle $\theta$ before momentarily stopping. Find $\theta$.

Multi-step problem.

A. Cons of energy to get speed of block $v_0$ before impact.

B. Cons of angular momentum (about $O$) during collision, where the system consists of block-plus-rod to get $L_1$ and $\omega_1$. The subscript 1 refers to immediately after the completely inelastic collision has taken place. The pivot point $O$ is selected as the reference so that its reaction forces exert zero torque.

C. Cons of energy to get how high it rises to get $\theta$.

A. $mgh = mv_0^2/2 \Rightarrow v_0 = \sqrt{2gh}$. Here, $m = 0.05$ kg and $h = 0.20$ m.

$v_0 = 1.98$ m/s.

B. $L_0 = bmv_0$ is the angular momentum of the system immediately before the collision. Here, $b$ is the length of the rod, to avoid confusion with the symbol $L$ for angular momentum.

$L_0 = 0.0396$ kg m$^2$/s.

$L_1 = I_{tot}\omega_1$. $I_{tot} = (1/3)Mb^2 + mb^2 = 0.01333$ kg m$^2$; and $L_1 = L_0$. So:

$\omega_1 = L_0/I_{tot} = 2.97$ rad/s.

C. $\Delta K + \Delta U_g = 0$. $\Delta K = 0 - (1/2)I_{tot}\omega_1^2 = -0.0588$ J.

$\Delta U_g = (Mg)(b/2)(1 - \cos \theta) + (mg)(b)(1 - \cos \theta)$. Equate to $-\Delta K$.

$(1 - \cos \theta)(m + M/2)(gb) = -0.0588$ J. Solve for $\cos \theta$ and $\theta$:

$\cos \theta = (1 - (0.0588 J)/(0.392 J)) = 0.850 \Rightarrow \theta = 31.8^\circ$.

If you work this out in algebra,

$$1 - \cos \theta = h/b \frac{m}{m + M/3} \frac{m}{m + M/2}.$$