High Resolution Laser Spectroscopy in Rubidium

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(Dated: April 29, 2013)

In this experiment we compare the predictions of quantum mechanics with experiment, with regards to the hyperfine splitting in rubidium 85 and 87. We characterize the doppler broadening in the atomic spectra of rubidium and find that the gas temperature is $300 \pm 30\, \text{k}$, which is expected. We show the resolution is some 100 times greater for doppler free saturation absorption spectroscopy than saturation absorption spectroscopy. Additionally we quantitatively confirm the predictions of the spectral lines of rubidium.

I. INTRODUCTION

Atomic spectra give intimate clues into the quantum structure of atoms, and is one of the most versatile phenomenon in physics. Spectra are unique to each atom, and have been used in a variety of ways, my favorite being the deduction of the composition of the sun. The 1981 Nobel Prize in physics was awarded to Arthur L. Schawlow, Nicolaas Bloembergen and Kai M. Siegbahn for their work on high resolution spectroscopy. With the development of lock-in amplifiers and saturation absorption spectroscopy, they could interrogate hyperfine structure. Understanding the hyperfine structure was a crucial step in developing quantum theory. The experiment section will outline the apparatus and details of the experiment, granting the reader a perspective of the results we hope to achieve. The theory section will delve into the analytic basis of the hyperfine structure, it will also include a discussion of doppler broadening and crossover resonance. We have devoted and entire section to saturation absorption spectroscopy, and it will be discussed in the section aptly named. The apparatus section will include a description of the apparatus which includes various optical elements along with the lock in amplifier. Finally there will be a discussion of results and conclusion.

II. EXPERIMENT

In this experiment we hope to measure the absorption spectrum for both isotopes of rubidium at 780.23nm. We expect through the use of saturation absorption spectroscopy and lock-in amplifiers to be able to reduce the effects of doppler broadening on our signal, thus increasing the resolution. The doppler broadening profile in the form of the full width half max of the uncorrected signal may be used to determine the temperature of the gas. The dips themselves will allow us to determine the frequency spacing and thus the energy levels of each isotope as well as the nuclear $g$ factor $g_I$. Two experiments will be undertaken to measure the spectrum of Rb at 780.23nm, both will succeed in giving us a spectrum, but one will be to find the doppler free spectrum. The results will be compared with calculated values from Fig. (1).

III. THEORY

A. Hyperfine Structure of Rb

The Hamiltonian that describes the hyperfine splitting of the spectra of an atom is described by

$$\mathcal{H} = A(I \cdot J),$$

where $I$ is the nuclear spin number and $J$ is the angular momentum of the electron. We also have that the total angular momentum of this system $F$ is:

$$F = I + J,$$

There is a $2F+1$ degeneracy of hyperfine states, as shown in Fig. (1). For the two isotopes of Rb, $I_{85} = \frac{5}{2}$ and $I_{87} = \frac{3}{2}$. We that the distribution of velocities for a given sample go as:

$$D(v)dv = \frac{m}{2\pi kT} e^{-\frac{mv^2}{2kT}} dv,$$
which is a Boltzmann distribution. Changing variables, so that \( \frac{\Delta \nu}{\nu_0} = \frac{v}{c} \), we can say:

\[
D(\nu) = \frac{c}{\nu_0} \sqrt{\frac{m}{2\pi kT}} e^{-\frac{mc^2\left(\frac{\nu}{\nu_0} - 1\right)^2}{2kT\nu_0^2}} \, d\nu,
\]

which is the distribution as a function of frequency, with \( \nu_0 \) being the resonant frequency. The full width half max, of a gaussian is as you might have guessed, the width of the curve at half the maximum value. Setting the exponent of eq. (4) to \( \ln(2) \) gives us:

\[
\frac{mc^2\left(\frac{\nu}{\nu_0} - 1\right)^2}{2\pi kT\nu_0^2} = \ln(2)
\]

\[
\frac{(\frac{\nu}{\nu_0} - 1)^2}{\nu_0^2} = \frac{2kT\ln(2)}{mc^2}
\]

\[
\Delta \nu = \sqrt{2\ln(2)k\nu_0^2T/m}.
\]

Eq. (5) tells us that the FWHM goes as the square root of temperature, which makes finding the temperature from the spectrum quite easy. We will discuss this in more detail a bit later.

**B. Doppler Broadening**

A large enough collection of atoms will obey Boltzmann statistics, so we can say:

\[
e^{-\frac{m \nu^2}{2kT}},
\]

where \( m \) is the mass of the atom, \( k \) is Boltzmann’s constant and \( T \) is the temperature. So long as there is a temperature the curve will have a certain broadness, meaning there are a variety of speeds. The doppler effect will then cause some atoms to perceive incident light as resonant, even if the light would not be resonant with an atom at rest. We then define the z-axis as the direction of the photon k-vector. Now \( \nu_1 \) is the resonant frequency for a transition and \( \nu_z \) is the z-axis component of the velocity, so

\[
\nu = \nu_1(1 + \frac{\nu_z}{c}),
\]

where \( c \) is the speed of light and \( \nu \) is the doppler shifted frequency. Eq. (7) gives that the observed frequency \( \nu \) is equal to the resonant frequency \( \nu_1 \) only when the z-component of the velocity \( \nu_z \) is zero. This makes sense, because that means the atom is neither moving toward or away from the beam and thus no doppler shift is observed. Now because there are a variety of velocities and directions, the observed spectrum will follow a Boltzmann scheme aswell, as seen in Fig. (2)[2]. Doppler broadening makes it very difficult to measure minute differences in the spectrum of rubidium as the broadening is several orders of magnitude greater than the width of the hyperfine operations themselves. Reducing the effect of doppler broadening will increase the effectiveness of our data.

**C. Saturated Absorption**

Saturation absorption spectroscopy reduces the effects of doppler broadening by isolating and recording only the light absorbed by atoms traveling perpendicular to the stimulating beam. Two laser beams of the same frequency are incident upon a gas of atoms in anti-parallel directions. These two beams are tuned to the resonant frequency. When the more intense “pump beam”, shines of the cell it excites a large number of atoms. The pump beam saturates the atoms such that the probe beam, which is an order of magnitude weaker, impinges on the same atoms, there are fewer to excite. The practical up-shot is that the probe beam does not collide with as many atoms, the saturated atoms become transparent to the probe beam. Thus when the probe beam impinges on the detector it more accurately shows the natural line width of the gas[9]. Figure 3 shows data gathered with saturated absorption spectroscopy.

**D. Crossover Resonance**

Hyperfine transitions are routinely smaller than the doppler broadening. As such atoms moving one way may be resonant with a hyperfine transition, at the same time
FIG. 3. Saturated absorption spectrum of Rb. Doppler broadening has been severely reduced by the pump beam, producing much a much more accurate spectrum with multiple distinct transitions.

FIG. 4. Crossover Resonance in Rb. The 4 dips correspond to absorption by each beam for blue-shifted and red-shifted frequencies due to the velocity distribution of atoms.

as atoms moving the other way which are resonant with another. This gives rise to cross over resonances, which are quite large in comparison to normal resonance dips. The basis for this was laid out in the section on doppler broadening. Figure 4 illustrates this phenomenon.

IV. APPARATUS

A. Lock-In Amplifier

A Lock-In amplifier allows us to eliminate a large amount of noise from a signal that reaches our detector. Less noise means a better signal to noise ratio, and thus resolution. The lock-in has two inputs: a reference beam and a signal beam. The pump beam is chopped at a certain frequency, and fed to the lock-in as the reference beam. The chopper is synchronised with the lock in. The signal beam is the same as before, just the probe beam incident on the PIN diode. The lock in compares the reference and signal beam, filtering out the noise. A low pass allows only the DC signal beam to pass. If the signal beam is out of phase with the reference beam, the output is null. Noise is by definition not phase dependent so a good amount is filtered out. The signal beam however is in phase with the reference beam and thus allowed through. This greatly improves the resolution of the system. A diagram of the lock-in amplifier can be seen in figure (5).

B. Setup

We use a tunable diode laser to sweep through the resonant frequency range for rubidium from the $^2S_\frac{1}{2}$ state to the $^2P_\frac{3}{2}$ state. A series of beam splitters and mirrors are used to reduce the probe beam power and establish counter prorogation in the rubidium cell. The probe beam then impinges on a PIN diode detector. With this setup we conduct two experiments. A schematic of the apparatus can be found in Fig. (6).

In the first experiment we must simply record the absorption spectra of the rubidium isotopes. This part does not require counter prorogating beams, one needs only to record the absorption spectrum from the PIN diode. The tunable diode is swept through the resonant band and both isotopes are found.

The saturation absorption spectrum experiment utilizes a fabry-perot etalon, lock in amplifier and numerous beam splitters. The etalon is used to determine where we are in a sweep among other things. After the beam passes through the etalon it impinges on a beamsplitter. The etalon is used to detect the light which passes in to it and sends this information to the lock in as the reference beam. This is explained in detail in the lock in section. The second splitter produces the pump and probe beams.
FIG. 6. Schematic of the experimental apparatus. The entire setup is used for the saturated absorption experiment, but the pump beam, chopper and etalon are not utilized for the simpler absorption spectrum experiment.

FIG. 7. Saturated absorption spectrum of $^{87}$Rb. The FWHM is 55MHz for the $F = 2 \rightarrow F' = 0$ transition corresponding to a temperature of 332.76K.

The probe beam is reduced in power and sent through the gas cell. The pump beam is chopped and sent in to the gas cell anti-parallel with the probe beam. The probe beam is recorded by the PIN diode.

V. RESULTS

Figures (7) and (8) show the spectra obtained using saturated absorption with a lock-in amplifier with a $\pi$ kHz chopping frequency, a 5mV sensitivity and 1ms time constant.

Figures (7) and (8) both have 6 peaks, which is more than we would expect given just the hyperfine transitions. The additional peaks can be explained as crossover resonances, and are marked in Fig. (9). The crossover resonance section explains their anomalous size. Using the temperature equation (5) we find from Fig. (V) and Fig. (V) that the temperature of the gas populations are 285.37K and 325.11K respectively. This averages to about 300K, which is room temperature as expected.

FIG. 8. Saturated absorption spectrum of $^{85}$Rb. The FWHM is 50MHz for the $F = 3 \rightarrow F' = 2$ transition corresponding to a temperature of 285.37K.

FIG. 9. Doppler free spectra for $^{85}$Rb and $^{87}$Rb.

VI. CONCLUSION

The doppler profiles for rubidium 85 and 87 seem to suggest a temperature of $285.37 \pm 30$ and $325.11 \pm 304$ respectively. We show good agreement between theory and experiment for the locations of the transitions for both isotopes as shown in the figures. We demonstrate a drastic increase in resolution through the use of saturation absorption spectroscopy. Lastly we deduce the nuclear g-factor’s $g_I$ for both isotopes to be $\frac{3}{2}$ for $^{87}$Rb and $\frac{5}{2}$ for $^{85}$Rb.
