Linear Algebra 2
Assignment # 10

Textbook Problems:

5B:1

Additional Problems:

1. Suppose $T \in \mathscr{L}(\mathbb{C}^4)$ where under the standard basis

$$\mathscr{M}(T) = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

   (a) Find all the eigenvalues of T.

   (b) Find a basis for all the eigenspaces.

   (c) Is T diagonalizable?

   (d) Find $\min(T)$.

2. Suppose $T \in \mathscr{L}(\mathbb{C}^3)$ where under the standard basis

$$\mathscr{M}(T) = \begin{bmatrix} -2 & 0 & 1 \\ -2 & -3 & 2 \\ -5 & -4 & 4 \end{bmatrix}$$

   Given that T has two eigenvalues $\lambda = 1$ and $\lambda = -1$. Find $\min(T)$.

3. Let $D \in \mathscr{L}(P_2(\mathbb{C}))$ be the derivative operator. Find $\min(D)$.

4. Let $T \in \mathscr{L}(V)$ be idempotent (see previous homework). Find all the possible polynomials $\min(T)$ can be. (Hint: There are three possibilities).

5. Suppose V is finite-diminutional and $T \in \mathscr{L}(V)$ define $\mathscr{A}_T \in \mathscr{L}(\mathscr{L}(V))$ by $\mathscr{A}_T(S) = TS$ for all $S \in \mathscr{L}(V)$. Prove that if $\lambda \in F$ is an eigenvalue of $\mathscr{A}_T$ then it is an eigenvalue of T.

   Notes:

   1. The other direction is true to but it is a little bit harder and I am not asking that.

   2. This question is abstract but not hard. You just have to keep in mind what all the definitions are and just don't get scared by the notation.