Linear Algebra 2
Assignment # 11

Textbook Problems:

Additional Problems:

1. Let $T \in \mathscr{L}(\mathbb{C}^4)$ such that

$$\mathscr{M}(T) = \begin{bmatrix} i & i & i & i \\ 0 & i & i & i \\ 0 & 0 & i & i \\ 0 & 0 & 0 & i \end{bmatrix}$$

   (a) Show $(x-i)^4$ annihilates T but $(x-i)^3$ does not.

   (b) What does this say about the eigenvalues of T?

   (c) Find $\min(T)$, make sure you explain how you know.

   (d) Is T diagonalizable?

2. Let $T \in \mathscr{L}(V)$ with distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_r$ and $p(x) = (x-\lambda_1)(x-\lambda_2)\ldots(x-\lambda_r)$. Show that if $v \in E_{\lambda_1} \oplus E_{\lambda_2} \oplus \ldots \oplus E_{\lambda_r}$ then $p(T)v = 0$.

3. Remember that if $A, B \in \mathbb{C}^{n\times n}$ then:

$$(A+B)_{i,j} = A_{i,j} + B_{i,j}$$

$$(AB)_{i,j} = \sum_{k=1}^{n} A_{i,k}B_{k,j}$$

$$A \text{ is upper-triangular if } A_{i,j} = 0 \text{ when } i > j$$

   For the following assume $A, B \in \mathbb{C}^{n\times n}$ are upper triangular and $T \in \mathscr{L}(V)$ with $\mathscr{M}(T) = A$ under some fixed basis. Also let $1 \le i \le n$ and $m > 0$.

   (a) Show $(AB)_{i,i} = A_{i,i}B_{i,i}$.

   (b) Prove by induction that $(A^m)_{i,i} = (A_{i,i})^m$.

   (c) Prove that if $p \in P(F)$ then $(p(A))_{i,i} = p(A_{i,i})$

   (d) Show that if $p \in P(F)$ annihilates T then $A_{i,i}$ is a root of p.

   (e) Show $A_{i,i}$ is an eigenvalue of T.

4. Let $T \in \mathscr{L}(\mathbb{C}^4)$ and

$$\mathscr{M}(T) = \begin{bmatrix} 2-2i & 2-4i & 4 & 2 \\ 0 & 3 & -2 & 3+4i \\ 0 & 0 & 2 & 1-4i \\ 0 & 0 & 0 & 6-4i \end{bmatrix}$$

   Find $\min(T)$.