Linear Algebra 2
Assignment # 12

Textbook Problems:

6A:12,20

Additional Problems:

1. For each of these norms give where they fail to satisfy the Parallelogram Equality. This shows these norms are not induced by an inner product.
   (a) $||\cdot||_1$ on $\mathbb{C}^3$.
   (b) $||\cdot||_\infty$ on $\mathbb{C}^3$.
   (c) $||\cdot||_1$ on $P_2(\mathbb{C})$ i.e. $||p||_1 = \int_0^1 |p|$.
2. Let $\mathbb{C}^3$ have the usual inner product. Show the following is an orthonormal basis.
$$\left(\frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{6}} \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right)$$
3. Let $V = C([-\pi, \pi], \mathbb{R})$ (set of real-valued continuous functions on $[-\pi, \pi]$) with inner product
$$\langle f, g \rangle = \int_{-\pi}^{\pi} fg.$$
   Show the following are an orthonormal list.
   (a) $\left(\frac{1}{\sqrt{2\pi}}, \sqrt{\frac{3}{2\pi^3}} x \right)$
   (b) $\left(\frac{2}{\sqrt{\pi}} \sin(x) \cos(x), \frac{1}{\sqrt{\pi}} \sin(x) \right)$
4. Let V be a real inner product space and $u, v \in V$. Show if $||u|| = ||v||$ then $u + v$ and $u - v$ are orthogonal.