Linear Algebra 2
Assignment # 13

Textbook Problems:

Additional Problems:

1. Apply the Gram-Schmidt process to the basis for $\mathbb{C}^4$:

$$\left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right)$$

   to obtain an orthonormal basis.

2. Suppose f is continuous on $[0,1]$ and

$$\int_0^1 f^2(x)\,dx = 1.$$

   Show that

$$\int_0^1 x f(x)\,dx \le \frac{1}{\sqrt{3}}.$$

3. Let

$$U = \operatorname{span}\left(\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 1 \\ 2 \\ 0 \end{bmatrix} \right).$$

   Find the projection of

$$w = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ -5 \\ -4 \\ -3 \\ -2 \\ -1 \end{bmatrix}$$

   on U.

4. Let V be a finite-dimentional inner product space and $U \subseteq V$ (note we are not assuming U is a subspace). Define the orthogonal complement of U as:

$$U^{\perp} = \{v \in V : \langle u, v \rangle = 0 \text{ for all } u \in U\}$$

Show that $U^{\perp}$ is a subspace of V.