Linear Algebra 2
Assignment # 14

Textbook Problems:

Additional Problems:

1. Let $n \in \mathbb{N}$. Show there exists $\delta_n \in P_n(\mathbb{R})$ such that for all $p \in P_n$,

$$\int_{-1}^{1} p(x)\delta_n(x) = p(0).$$

Note: δ_n behaves like the "Dirac delta function" δ, which puts infinite weight on 0 and zero weight everywhere else so for any continuous f when you integrate the function $f \cdot \delta$ on an interval the only value of f that maters is its value at 0. This works great in a finite dimentional space like P_n. Unfortunately, (for my Real II people) $\{\delta_n\}$ is not L^2-Cauchy so there is no δ on $[-1, 1]$ such that $\delta_n \to \delta$. So there is no Dirac delta function that works every continuous function f. However that doesn't prevent physicists from pretending that δ exists.

2. Using the setup of the previous problem find δ_1.

3. Let $\mathbb{C}^n$ have the usual inner product. Let $T \in (\mathbb{C}^2, \mathbb{C}^3)$ be defined by:

$$T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} 2ia + (3+2i)b \\ 2a \\ ib \end{bmatrix}$$

Find $T^*\left(\begin{bmatrix} 1 \\ i \\ 1+i \end{bmatrix}\right)$.

4. Remember the trace from Linear I (I sure do). If $A \in F^{n\times n}$ then $\text{tr}(A) = \sum_{i=1}^{n} A_{i,i}$. Prove the following facts (from Linear I)

   (a) The trace is a linear functional

   (b) If $A, B \in F^{n\times n}$ then $\text{tr}(AB) = \text{tr}(BA)$

   (c) If A and B are similar (there exists an invertible $C \in F^{n\times n}$ such that $A = CBC^{-1}$), then $\text{tr}(A) = \text{tr}(B)$.
   Note: A and B are similar if they are matrices of the same transformation $T \in \mathscr{L}(F^n)$ with respect to different basis