Linear Algebra 2
Assignment # 5

Textbook Problems:

3B: 5,6,20
3C: 4

Additional Problems:

Note in these problems you do not need to show any of these maps are linear.

Note this has been updated for the new definition of P_n (on 2/23/26 at 2:44 PM)

1. Let $T \in \mathcal{L}(\mathbb{R}^{2\times 2}, P_3(\mathbb{R}))$ and $S \in \mathscr{L}(P_3(\mathbb{R}), \mathbb{R}^3)$ be defined by:

$$S(ax^3 + bx^2 + cx + d) = \begin{bmatrix} a+b \\ a+c \\ a+d \end{bmatrix}$$

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+b+c)x^3 + (a+b)x^2 + dx + c$$

   (a) Find a basis for $\mathrm{Nul}(T)$ and $\mathrm{Rg}(T)$.
   (b) Find a basis for $\mathrm{Nul}(S)$ and $\mathrm{Rg}(S)$.
   (c) Find $(ST)\left(\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}\right)$
   (d) Find $\mathscr{M}(S)$ and $\mathscr{M}(T)$ using the standard bases.

2. Let U, V, W are finite-dimensional vector spaces. Let $S \in \mathscr{L}(V, W)$ and $T \in \mathscr{L}(U, V)$. Show the following:
   (a) $\mathrm{Nul}(T) \leq \mathrm{Nul}(ST)$
   (b) $\mathrm{Rg}(ST) \leq \mathrm{Rg}(S)$
   (c) $\mathrm{rank}(ST) \leq \min(\mathrm{rank}(S), \mathrm{rank}(T))$

3. Let $V = P_2(\mathbb{R})$ and $W = \mathbb{R}^3$ and $T \in \mathscr{L}(V, W)$ be defined by

$$T(ax^2 + bx + c) = \begin{bmatrix} a+b \\ 2a+2b \\ 3a+3b \end{bmatrix}$$

   Find bases for $\mathscr{U}_V$ and $\mathscr{U}_W$ for V and W such that:

$$\mathscr{M}(T, \mathscr{U}_V, \mathscr{U}_W) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$