Linear Algebra 2
Assignment # 7

Textbook Problems:

4: 1,5

Additional Problems:

1. Let $f(x) = x^5 - 4x^4 + 7x^3 - 13x^2 + 7x - 8$ and $s(x) = x^3 - 4x^2 + 6x - 9$.
   (a) Find $q, r \in P(\mathbb{R})$ such that $f(x) = s(x)q(x) + r(x)$ and $\deg(r) < \deg(s)$.
   (b) Find $s(3)$.
   (c) Use this to find $f(3)$

2. Let $p \in P(\mathbb{R})$. Remember $\lambda \in \mathbb{R}$ is said to be a root of multiplicity $m \geq 1$ if there exists $s \in P$ such that $p(x) = (x - \lambda)^m s(x)$ and $s(\lambda) \neq 0$. You can use your calculus knowledge for these problems.
   (a) In the definition above I had $m \geq 1$. This is usually how it is used, but show that you could remove this limitation and a non-root is just a root of multiplicity 0.
   (b) Show that λ is a root of multiplicity 2 if and only if $p(\lambda) = p'(\lambda) = 0$ and $p''(\lambda) \neq 0$.
   (c) Make a similar characterization for multiplicity $m \geq 1$. You do not need to prove this.

3. Using:

$$\begin{aligned} &x^7 - \frac{142}{21}x^6 + \frac{27521}{1764}x^5 - \frac{4588}{441}x^4 - \frac{67099}{1764}x^3 + \frac{122699}{882}x^2 - \frac{21428}{147}x + \frac{32708}{441} \\ =&(x - 3 - \frac{1}{2}i)(x - \frac{5}{7} - 2i)(x - \frac{2}{3} + \frac{2}{3}i)(x + 2)\left(x^3 - (\frac{92}{21} - \frac{11}{6}i)x^2 + (\frac{37}{7} - \frac{233}{42}i)x - 5 + \frac{73}{21}i\right) \end{aligned}$$

   Factor the following into irreducible polynomials in $P(\mathbb{C})$:

$$x^3 - (\frac{92}{21} - \frac{11}{6}i)x^2 + (\frac{37}{7} - \frac{233}{42}i)x - 5 + \frac{73}{21}i$$