Linear Algebra 2
Assignment # 8

Textbook Problems:

5A: 1,2

Additional Problems:

1. In this question we remember that we can also evaluate a polynomial at an $n \times n$ matrix. Consider:

$$A = \begin{bmatrix} 7 & -9 \\ 6 & -8 \end{bmatrix}$$

   (a) Let $p(x) = x^2 + 1$. Calculate $p(A)$.

   (b) We have not talked about characteristic polynomials but go back to your linear algebra notes and figure out how to compute $g(x) = \text{char}(A)$.

   (c) Compute $g(A)$.

2. Let V be a vector space and $T \in \mathscr{L}(V)$ and $g \in P(F)$.

   (a) Show if $g(0) = 0$ then $\text{Nul}(T) \subseteq \text{Nul}(g(T))$.

   (b) Show if $g(0) \neq 0$ then $\text{Nul}(T) \cap \text{Nul}(g(T)) = \{0\}$.

3. Let V be a vector space and $T, S \in \mathscr{L}(V)$ and $g, h \in P(F)$.

   (a) Show if S and T commute (i.e. $ST = TS$) then $\text{Rg}(S)$ is invariant under T.

   (b) Show that $\text{Rg}(g(T))$ is invariant under $h(T)$.

   (c) Show that $\text{Rg}(g(T))$ is invariant under T.