Linear Algebra 2
Assignment # 9

Textbook Problems:
5A: 5, 6, 7

Additional Problems:

1. Let $P \in \mathscr{L}(V)$ we say that P is an **idempotent operator** if $P^2 = P$
   (a) Show that if λ is an eigenvalue of an idempotent operator P then either $\lambda = 0$ or $\lambda = 1$.
   (b) Let $T \in \mathscr{L}(\mathbb{C}^{2\times 2})$ be a linear transformation defined by:
   $$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a & b+c \\ 0 & 2a \end{bmatrix}.$$
   Show that T is idempotent.
   (c) Find $T^{302}\left(\begin{bmatrix} 3i & 5+2i \\ \pi^2+1 & 3i-4 \end{bmatrix}\right)$.
   (d) Find E_0 and E_1, the eigenspaces for T associated with $\lambda = 0$ and $\lambda = 1$ respectively.

2. Operators $T, S \in \mathscr{L}(V)$ are said to be similar if there exists an invertible $R \in \mathscr{L}(V)$ such that $T = RSR^{-1}$. Show that similar operators have the same eigenvalues.

3. Let $R \in \mathscr{L}(V)$ be invertible. Show that if λ is an eigenvalue of R, then $\lambda \neq 0$ and $\frac{1}{\lambda}$ is an eigenvalue of R^{-1}.

4. Let V be a vector space, $T \in \mathscr{L}(V)$ have eigenvalue $\lambda \in F$, and corresponding eigenvector $v \in V$.
   (a) Use induction (good practice) to prove for all $k \in \mathbb{N}$, λ^k is an eigenvalue of T^k with corresponding eigenvector v.
   (b) Prove (you can just do a ... type argument for this) that if $p \in P(F)$ then $p(T)v = p(\lambda)v$.