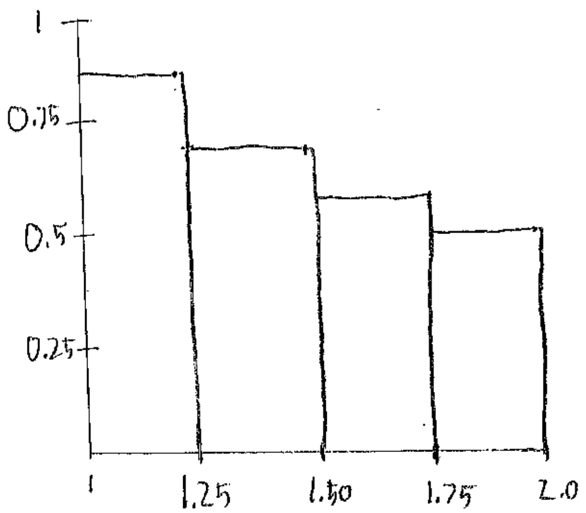
5.1 Review Solutions

Luke Ellersick

3) Setup

a) $f(x) = 1/x$ from $x=1$ to $x=2$

4 rectangles $\Delta x = \frac{2-1}{4} = 0.25$




$f(1.25) = 0.8$

$f(1.50) = 0.6667$

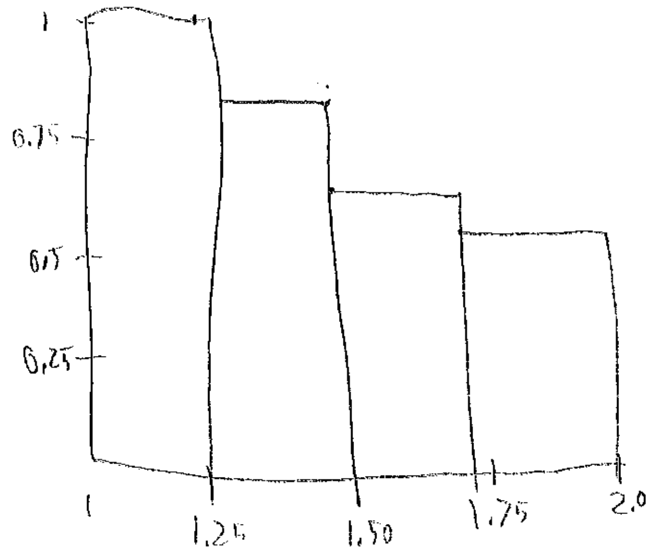
$f(1.75) = 0.5714$

$f(2.00) = 0.5$

$R_4 = 0.6346$

underestimate

b)




$f(1) = 1$

$f(1.25) = 0.8$

$f(1.5) = 0.6667$

$f(1.75) = 0.5714$

$L_4 = 0.7596$

5.1 Review Solution

Luke Ellersick

13 $\frac{5-2}{6} = \frac{1}{2}$

$x_i = 2 + \left(i - \frac{1}{2}\right) \Delta x$

$i = 3$

$x_3 = 2 + \left(3 - \frac{1}{2}\right) \cdot \frac{1}{2}$

$= 2 + \frac{5}{2} \cdot \frac{1}{2}$

$= 2 + \frac{5}{4}$

$\frac{13}{4}$

$f(x) = x^2 + 2$

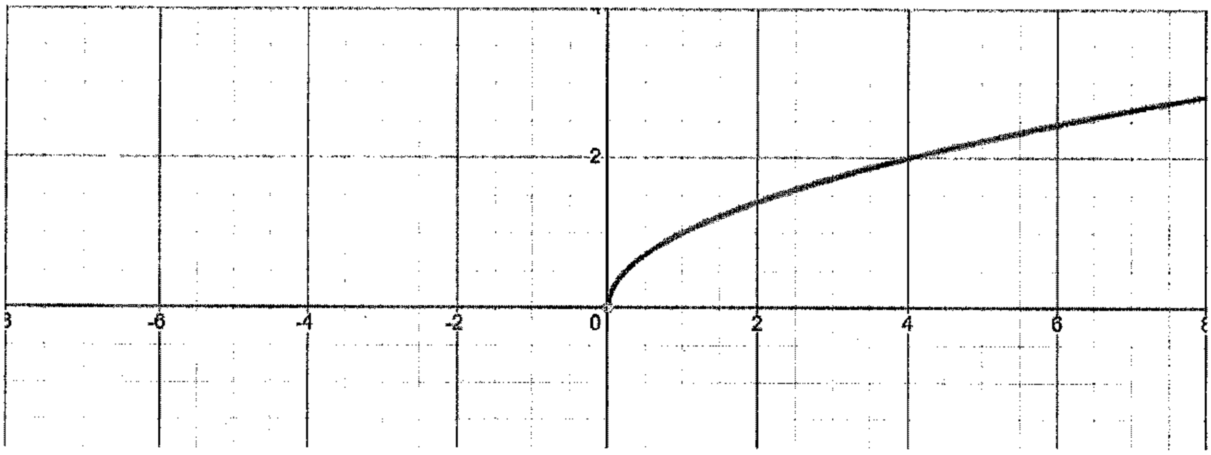
$f(x_3^*) = \left(\frac{13}{4}\right)^2 + 2$

$= \frac{169}{16} + 2$

$\boxed{\frac{201}{16}}$

5.1

The graph $f(x) = \sqrt{x}$ is shown below.




Use the 6 rectangle to find the estimate of the area bounded by the graph and the lines x=0 and x=6

7. Find $L_6$

Answer: 8.382

8. Find $M_6$

Answer: 9.850

9. Find $R_6$

Answer: 10.832

Ava Mertens 5.3 Solutions

1) $\int_0^2 (2x-3)(4x^2+1)\,dx$

$(2x-3)(4x^2+1) = 8x^3 - 12x^2 + 2x - 3$

Evaluate from 0 to 2:

$2(2^4) - 4(2^3) + (2^2) - 3(2) = 32 - 32 + 4 - 6 = \boxed{-2}$

2) $\int \sec t(\sec t + \tan t)\,dt$

$\sec t(\sec t + \tan t) = \sec^2 t + \sec t \tan t$

$\frac{d}{dt}(\tan t) = \sec^2 t$ $\frac{d}{dt}(\sec t) = \sec t \tan t$

$\int \sec t\,dt = \tan t$ $\int \sec t \tan t\,dt = \sec t$

$\boxed{\tan t + \sec t + C}$

5.3

Emiliano Martinez

5. $\int_0^2 (2x-3)(4x^2+1)\,dx$

expand

$= \int_0^2 (8x^3-12x^2+2x-3)\,dx$

$= \left[8\cdot\frac{x^4}{4} - 12\cdot\frac{x^3}{3} + 2\cdot\frac{x^2}{2} - 3x\right]_0^2$

$= \left[2x^4 - 4x^3 + x^2 - 3x\right]_0^2$

$= (2(2)^4 - 4(2)^3 + (2)^2 - 3(2))$

$- (2(0)^4 - 4(0)^3 + (0)^2 - 3(0))$

$= -2$

5.3

Emiliano Martinez

54. $\int (2+\tan^2\theta)\,d\theta$

$= \int 2\,d\theta + \int \tan^2\theta\,d\theta$

1. $\int 2\,d\theta = 2\theta$

trig identity

2. $\int \tan^2\theta\,d\theta = \int (\sec^2\theta - 1)\,d\theta$

$= \int \sec^2\theta\,d\theta - \int 1\,d\theta = \tan\theta - \theta$

then

$2\theta + (\tan\theta - \theta) + C$

$= \theta + \tan\theta + C$

1. $f(x) = \int_1^x \frac{\ln t}{t^2}\,dt$, find $f'(x)$

plug in x

since upper limit is x

$f'(x) = \frac{\ln x}{x^2}$

2. $f(x) = \int_9^{x^3} \frac{\ln t}{t^2}\,dt - \int_9^x \frac{\ln t}{t^2}\,dt$

$f'(x) = \frac{\ln(x^3)}{(x^3)^2} \cdot 3x^2 - \frac{\ln x}{x^2} = \frac{3\ln x}{x^6} \cdot 3x^2 - \frac{\ln x}{x^2}$

$= \frac{9\ln x}{x^4} - \frac{\ln x}{x^2}$

$f'(x) = \frac{\ln x(9-x^2)}{x^4}$

Rowan Haney

5.4 solutions

# Solutions 5.5

1. $\int 2x\sqrt{1+x^2}\,dx$ Let $u=1+x^2$, $du=2x\,dx$, $dx=\frac{du}{2x}$

$=\int 2x\sqrt{u}\,dx = \int 2x\sqrt{u}\,\frac{du}{2x}$

$=\int \sqrt{u}\,du = \frac{2}{3}u^{3/2}+C = \frac{2}{3}(1+x^2)^{3/2}+C$

---

2. $\int \frac{1}{x^2(x^4+1)^{3/4}}\,dx$

$u=x^{-4}$
$u^{-\frac{1}{4}}=x$
$-\frac{1}{4}u^{-\frac{5}{4}}\,du=dx$
$u^{-1}=x^4$
$u^{-\frac{1}{4}}=x$
$x^2=u^{-\frac{1}{2}}$

$=\int -\frac{1}{4}\,\frac{1}{u^{5/4}\;u^{-\frac{1}{2}}(u^{-1}+1)^{3/4}}\,du$

$=-\frac{1}{4}\int \frac{1}{u^{\frac{3}{4}}\,u^{-\frac{1}{2}}(u^{-1}+1)^{3/4}}\,du$

$=-\frac{1}{4}\int \frac{du}{(u(u^{-1}+1))^{3/4}}$

$=-\frac{1}{4}\int \frac{du}{(1+u)^{3/4}}$ $v=1+u$, $dv=du$

$-\frac{1}{4}\int \frac{dv}{v^{3/4}} = \frac{v^{1/4}}{1/4} = 4v^{1/4}$

$=-\frac{1}{4}\cdot 4\cdot v^{1/4}+C = -(1+u)^{1/4}+C$

$=-(1+x^{-4})^{1/4}+C$

Cooper Cross

# 5.5 U-Sub Solutions

Caleb [illegible]
Calc 2 section 3

1) $\int x\sqrt{x+2}$  $u = x+2$
$du = dx$

$\int (u-2)\sqrt{u}\,du \to (u-2)u^{1/2}$

$= \int u^{3/2} - 2u^{1/2}\,du \to \frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + C$

$\boxed{\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C}$

2) $\int \cos(1+5t)\,dt$  $u = 1+5t$
$\frac{du}{5} = \frac{5\,dt}{5} \to \frac{1}{5}du = dt$

$\int \cos(u)\cdot\frac{1}{5}du = \frac{1}{5}\int \cos(u)\,du \to \frac{1}{5}\sin(u)+C$

$\boxed{\frac{1}{5}\sin(1+5t)+C}$

**5.6 Integration by Parts → Solutions**
**Kamryn Harris**

**Question 1**

$$t = \ln(2x)$$
$$e^t = 2x$$
$$e^t dt = 2\,dx$$
$$\frac{e^t}{2} dt = dx$$

① $\int \sin(\ln(2x))\,dx \implies$

$= \int \sin(t) \cdot \frac{e^t}{2}\,dt$

$= \frac{1}{2}\int e^t \sin(t)\,dt$

$I = \int e^t \sin(t)\,dt$ $\qquad u = \sin(t) \quad dv = e^t dt$ $\qquad du = \cos(t)dt \quad v = e^t$

$= e^t \sin(t) - \int e^t \cos(t)\,dt$

$\int e^t \cos(t)\,dt$ $\qquad u = \cos(t) \quad dv = e^t dt$ $\qquad du = -\sin(t)dt \quad v = e^t$

$= e^t \cos(t) - \int e^t(-\sin(t))\,dt$

$= e^t \cos(t) + \int e^t \sin(t)\,dt$

$= e^t \cos(t) + I$

$I = e^t \sin(t) - (e^t \cos(t) + I)$

$I = e^t(\sin(t) - \cos(t)) - I$

$2I = e^t(\sin(t) - \cos(t)) + C \implies I = \frac{1}{2} e^t (\sin(t) - \cos(t)) + C$

$\int e^t \sin(t)\,dt = \frac{1}{2} e^t (\sin(t) - \cos(t)) + C$

$$\begin{aligned}\int \sin(\ln(2x))\,dx &= \frac{1}{2}\int e^t \sin(t)\,dt \\ &= \frac{1}{2}\left[\frac{1}{2} e^t (\sin(t) - \cos(t)) + C\right] \\ &= \frac{1}{4} e^t (\sin(t) - \cos(t)) + C \\ &= \frac{1}{2} x\left[\sin(\ln(2x)) - \cos(\ln(2x))\right] + C\end{aligned}$$

**Question 2**

② $\int_0^{\frac{\pi}{3}} \sin(x)\ln(\cos(x))\,dx \implies$ $u = \cos(x)$

$du = -\sin(x)dx$

$-du = \sin(x)dx$

$= \int_1^{\frac{1}{2}} \ln(u) \cdot -du$

$= -\int_{\frac{1}{2}}^{1} \ln(u) \cdot -du$

$= \int_{\frac{1}{2}}^{1} \ln(u)\,du$

$= \left[ u\ln(u) - u \Big|_{\frac{1}{2}}^{1} \right]$

$= \left[((1)\ln(1) - 1) - ((\tfrac{1}{2})\ln(\tfrac{1}{2}) - \tfrac{1}{2})\right]$ $\longrightarrow$ $\ln(\tfrac{1}{2}) = \ln(1) - \ln(2)$

$= 0 - \ln(2)$

$= -\ln(2)$

$= \left[(-1) - (-\tfrac{1}{2}\ln(2) - \tfrac{1}{2})\right]$

$= \left[-1 + \tfrac{1}{2}\ln(2) + \tfrac{1}{2}\right]$

$= -\tfrac{1}{2} + \tfrac{1}{2}\ln(2) = \tfrac{1}{2}(\ln(2) - 1)$

By: Mia Rush

# 5.6: Partial Fractions Key

1) $\int \frac{5x-13}{x^2-5x+6}\,dx$

$= \int \frac{5x-13}{(x-3)(x-2)}\,dx$

$\frac{5x-13}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x-2}$

$5x-13 = A(x-2) + B(x-3)$

let $x = 2$

$5(2)-13 = A(-2+2) + B(-2+3)$

$10-13 = 0A + 1B$

$-3 = B$

let $x = 3$

$5(3)-13 = A(3-2) + B(3-3)$

$15-13 = 1A + 0B$

$2 = A$

$\int \frac{2}{x-3}\,dx + \int -\frac{3}{x-2}\,dx$

$\boxed{= 2\ln|x-3| - 3\ln|x-2| + C}$

2) $\int \frac{x}{(x-1)(x-2)^2}\,dx$

$= \int \frac{A}{x-1}\,dx + \int \frac{B}{x-2}\,dx + \int \frac{C}{(x-2)^2}\,dx$

$\frac{x}{(x-1)(x-2)^2} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

$x = A(x-2)^2 + B(x-1)(x-2) + C(x-1)$

let $x = 2$

$2 = A(0) + B(1)(0) + C(1)$

$2 = C$

let $x = 1$

$1 = A(1) + B(0)(-1) + C(0)$

$1 = A$

let $x = 3$ (+ plug in A + B)

$3 = 1(1) + B(2)(1) + 2(2)$

$3 = 1 + 2B + 4$

$3 = 5 + 2B$

$-2 = 2B$

$-1 = B$

$= \int \frac{1}{x-1}\,dx + \int \frac{-1}{x-2}\,dx + \int \frac{2}{(x-2)^2}\,dx$

$u = x-2$

$du = dx$

$\int \frac{1}{u^2}\,du = \int u^{-2}\,du = -u^{-1} = -\frac{1}{u}$

$= -\frac{1}{x-2}$

$2\int \frac{1}{u^2} = -\frac{2}{x-2}$

$\boxed{= \ln|x-1| - \ln|x-2| - \frac{2}{x-2} + C}$

# 5.6 Intergration by Parts

LUCAN R.

$\boxed{uv - \int v\,du}$ key

1

$= \int xe^x\,dx \longrightarrow = xe^x - \int e^x\,dx$

$u = x \quad du = dx$

$v = e^x \quad dv = e^x\,dx$

$= xe^x - e^x + C$

$\boxed{= e^x(x-1) + C}$

2.

$= \int x^2 \ln(x)\,dx$

$\rightarrow \frac{x^3}{3}\ln(x) - \int \frac{x^3}{3}\cdot\frac{1}{x}\,dx$

$u = \ln(x) \quad du = \frac{1}{x}\,dx$

$dv = x^2\,dx \quad v = \frac{x^3}{3}$

$= \frac{x^3}{3}\ln(x) - \int \frac{x^2}{3}\,dx =$

$\frac{x^3}{3}\ln(x) - \frac{1}{3}\int x^2\,dx =$

$\frac{x^3}{3}\ln(x) - \frac{1}{3}\cdot\frac{x^3}{3} + C$

$\boxed{= \frac{x^3}{3}\ln(x) - \frac{x^3}{9} + C}$

1) $\int \frac{\sqrt{x^2-1}}{x^4}dx$

Noelle sec:02

$x = \sec\theta$

$dx = \sec\theta\tan\theta\, d\theta$

$$\int \frac{\sqrt{\sec^2\theta-1}}{\sec^4\theta}(\sec\theta\tan\theta)d\theta$$

$$\int \frac{\tan\theta\cdot\tan\theta}{\sec^3\theta}d\theta$$

$$\int \frac{\tan^2\theta}{\sec^3\theta}d\theta = \frac{\sin^2\theta}{\cos^2\theta}\cdot\cos^3\theta = \sin^2\theta\cos\theta$$

$U = \sin\theta$

$dv = \cos\theta\, d\theta$

$$\int u^2 du = \frac{1}{3}u^3 + c = \frac{1}{3}\sin^3\theta + c$$

$X = \sec\theta$

$\cos\theta = \frac{1}{x}$

$$\sin\theta = \frac{\text{oppo}}{\text{hypo}} = \frac{\sqrt{x^2-1}}{x} = \boxed{\frac{1}{3}\left(\frac{\sqrt{x^2-1}}{x}\right)^3 + C}$$

2) $\int \frac{1}{\sqrt{x^2+9}}dx$

$x = 3\tan\theta$

$dx = 3\sec^2\theta\, d\theta$

$$\int \frac{1}{\sqrt{(3\tan\theta)^2+9}}\cdot(3\sec^2\theta)d\theta$$

$$\int \frac{1}{\sqrt{9\tan^2\theta+9}}\cdot(3\sec^2\theta)d\theta$$

$$= \sqrt{9(\underbrace{\tan^2\theta+1}_{\sec^2\theta})}$$

$$= \sqrt{9\sec^2\theta} = 3\sec\theta$$

$\tan\theta = \frac{x}{3}$

$\sec\theta = \frac{\text{hypo}}{\text{adj}} = \frac{\sqrt{x^2+9}}{3}$

$$\int \frac{3\sec^2\theta}{3\sec\theta}d\theta$$

$$= \int \sec\theta\, d\theta \longrightarrow \ln|\sec\theta + \tan\theta| + C$$

$$\boxed{\ln\left|\frac{\sqrt{x^2+9}}{3} + \frac{x}{3}\right| + C}$$

# 5.7 Review Proplem Solution

Omar Al-Omran

1. $\sin^2 x = 1 - \cos^2 x$

$u = \cos x$
$du = -\sin x\, dx$

$\sin^3 x \cos^2 x = (\sin^2 x)(\sin x)\cos^2 x$
$= (1-\cos^2 x)(\sin x)\cos^2 x$

$\int (1-u^2)u^2(-du)$
$= -\int (u^2 - u^4)\, du$

$= -\left(\frac{u^3}{3} - \frac{u^5}{5}\right)$

$= -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + c$

2. $\frac{5x+1}{(2x+1)(x-1)} = \frac{A}{2x+1} + \frac{B}{x-1}$

$5x+1 = A(x-1) + B(2x+1)$

let $x = 1$ | let $x = -1/2$
$6 = 3B$ | $-3/2 = -3/2 A$
$B = 2$ | $A = 1$

$\int \frac{5x+1}{(2x+1)(x-1)}\, dx = \int \frac{1}{2x+1}\, dx + \int \frac{2}{x-1}\, dx$

$1/2 \ln|2x+1| + 2\ln|x-1| + c$

① Section 5.7 Problem 6 (Easy) Jennaya Horne

$$\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx$$

We know : $\sin^2 x \cos^2 x = (\sin x \cos x)^2$

$\sin x \cos x = \frac{1}{2}\sin(2x) \rightarrow$ Trig identity

so ,

$$\sin^2 x \cos^2 x = \frac{1}{4}\sin^2(2x)$$

$$\frac{1}{4}\int_0^{\pi/2} \sin^2(2x) \, dx$$

$\sin^2 x = \frac{1-\cos(2x)}{2} \rightarrow$ Trig identity

So

$$\sin^2(2x) = \frac{1-\cos(4x)}{2}$$

$$\frac{1}{4}\int_0^{\pi/2} \frac{1-\cos(4x)}{2} \, dx = \frac{1}{8}\int_0^{\pi/2} 1-\cos(4x) \, dx$$

$u = 4x$

$du = 4 \, dx$

$\frac{1}{4} du = dx$

$$\frac{1}{4}\int \cos(u) \, du$$

$$= \frac{\sin(u)}{4}$$

$$= \frac{\sin(4x)}{4}$$

so

$$\frac{1}{8}\left(x - \frac{\sin(4x)}{4}\right)\Big]_0^{\pi/2}$$

$$\frac{1}{8} \cdot \frac{\pi}{2} = \boxed{\frac{\pi}{16}}$$

(2) Section 5.7 Problem 18 (Hard) Jennaya Horne

$$\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{16-x^2}}\,dx$$

$$\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{4^2-x^2}}\,dx$$

$\cos^2\theta = 1-\sin^2\theta$

$x^2 = (4\sin\theta)^2$ $\qquad x^3 = (4\sin\theta)^3$

Since $x = 4\sin\theta$

$dx = 4\cos\theta$

$$\sqrt{16-x^2} = \sqrt{16-16\sin^2\theta}$$

$$= \sqrt{16(1-\sin^2\theta)}$$

$$= \sqrt{16\cos^2\theta}$$

$$= 4\cos^2\theta$$

So

$$\int_0^{2\sqrt{3}} \frac{64\sin^3\theta}{4\cancel{\cos^2\theta}}\,4\cancel{\cos\theta}\,d\theta$$

$$\int_0^{2\sqrt{3}} 64\sin^3\theta\,d\theta$$

change bounds

$x=0$ so $0 = 4\sin\theta$

$\theta = 0$

$x = 2\sqrt{3} = 4\sin\theta$

$$\sin\theta = \frac{2\sqrt{3}}{4_2} = \frac{\sqrt{3}}{2}$$

$$\theta = \pi/3$$

So $64\int_0^{\pi/3} \sin^3\theta\,d\theta$

⟶ next page

$64\int \sin^3\theta \, d\theta$

Jennaya Horne

$$64\int_0^{\pi/3} \sin^2\theta \cdot \sin\theta \, d\theta$$

$$64\int_0^{\pi/3} (1-\cos^2\theta)\sin\theta \, d\theta$$

$u = \cos\theta$

$du = -\sin\theta \, d\theta$

$-du = \sin\theta \, d\theta$

$$64 \cdot -1\int_0^{\pi/3} (1-u^2)\, du$$

$\theta = \pi/3$

$$= 64\left(-u + \frac{u^3}{3}\right)$$

$$= 64\left[-\cos\theta + \frac{\cos^3\theta}{3}\right]_0^{\pi/3}$$

$$= \left(\frac{1}{2} - \frac{(1/2)^3}{3}\right)$$

$$= \left(\frac{1}{2} - \left(\frac{1}{8}\cdot\frac{1}{3}\right)\right)$$

$$= \left(\frac{1}{2} - \frac{1}{24}\right)$$

$$= \left(-\frac{11}{24}\right)$$

$$= \frac{11}{24}$$

$\theta = 0$

$\cos(0) = 1$

$$\left(-1 + \frac{1^3}{3}\right)$$

$$= (-1 + 1/3)$$

$$= (-2/3)$$

Top - Bottom

$$-\frac{11}{24} + \frac{2}{3}$$

$$= -\frac{11}{24} + \frac{16}{24}$$

$$= \frac{5}{24} \cdot 64$$

$$= \boxed{\frac{40}{3}}$$

a). Identify factors in denominator: $(x+3)$, $(3x+1)$
For every factor, create a fraction with a constant like A or B

$$\boxed{\frac{2x}{(x+3)(3x+1)} = \frac{A}{x+3} + \frac{B}{3x+1}}$$

b). Factor denominator of $\frac{1}{x^3+2x^2+x}$

$x^3+2x^2+x = x(x^2+2x+1)$

Perfect square so: $x(x^2+2x+1) = x(x+1)^2$

Must include a fraction for every power, so:

$$\boxed{\frac{1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}}$$

$$\frac{5x+1}{(2x+1)(x-1)} = \frac{A}{2x+1} + \frac{B}{x-1}$$

$$5x+1 = A(x-1) + B(2x+1)$$

Find B, let $x=1$

$$5(1)+1 = A(1-1) + B(2(1)+1)$$

$$6 = 0 + 3B$$

$$B = \frac{6}{3} = 2$$

Find A, let $x = -\frac{1}{2}$

$$5(-\tfrac{1}{2})+1 = A(-\tfrac{1}{2}-1) + B(2(-\tfrac{1}{2})+1)$$

$$-\frac{5}{2} + \frac{2}{2} = A(-\tfrac{3}{2}) + 0$$

$$-\frac{3}{2} = -\frac{3}{2}A$$

$$A = 1$$

Rewrite as: $\frac{1}{2x+1} + \frac{2}{x-1}$

$$\int \left(\frac{1}{2x+1} + \frac{2}{x-1}\right) dx$$

$$\int \frac{1}{2x+1}\,dx + \int \frac{2}{x-1}\,dx$$

For $\int \frac{1}{2x+1}\,dx$ use u-sub where $u = 2x+1$ and $du = 2\,dx$

$$\frac{1}{2}\ln|2x+1|$$

$$\int \frac{2}{x-1}\,dx = 2\ln|x-1|$$

$$\boxed{\frac{1}{2}\ln|2x+1| + 2\ln|x-1| + C}$$

Tatiana Montoya Section 2
5.7 trig integrals

① $\int \sin^3(x)\cos^2(x)\, dx$

↳ $\sin^2(X)\sin(X)$

$$\int \underbrace{\sin^2(x)}_{1-\cos^2(x)} \cos^2(X)\sin(x)\, dx$$

$$\int (1-\cos^2(x))\cos^2(X)\sin(X)\, dx$$

$U=\cos(X) \quad du=-\sin(x)\,dx$

$$-\int(1-u^2)u^2\, du \rightarrow -\int(u^2-u^4)\, du$$

$$-\left(\frac{u^3}{3}-\frac{u^5}{5}\right)+c \rightarrow \frac{-\cos^3(x)}{3}+\frac{\cos^5(x)}{5}+C$$

$$\boxed{\frac{\cos^5(x)}{5}-\frac{\cos^3(x)}{3}+C}$$

Tatiana Montoya
Section 2
5.7 trig integrals

② $\int \sin^4(x)\cos^2(x)\,dx$

$\sin^2(x) = \dfrac{1-\cos(2x)}{2}$ $\cos^2(x) = \dfrac{1+\cos(2x)}{2}$

$\int \sin^4(x)\cos^2(x)\,dx = \int (\sin^2(x))^2 \cos^2(x)\,dx$

$\int \left(\dfrac{1-\cos(2x)}{2}\right)^2 \left(\dfrac{1+\cos(2x)}{2}\right) dx$

$\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) \to \frac{1}{8}$

$\dfrac{1}{8}\int (1-\cos(2x))^2 (1+\cos(2x))$

$(1-\cos(2x))^2 = 1-2\cos(2x) + \cos^2(2x)$

multiply by $(1+\cos(2x))$

$(1-2\cos(2x) + \cos^2(2x))(1+\cos(2x))$

$(1+\cos(2x)) + (-2\cos(2x) - 2\cos^2(2x)) + (\cos^2(2x) + \cos^3(2x))$

$1-\cos(2x) - \cos^2(2x) + \cos^3(2x)$

$\dfrac{1}{8}\int [1-\cos(2x) - \cos^2(2x) + \cos^3(2x))\,dx$

① $\int 1\,dx = x$

③ $-\int \cos^2(2x) = -\int \dfrac{1+\cos(4x)}{2}$

② $\int -\cos(2x)\,dx = \frac{1}{2}\sin(2x)$

$= \dfrac{x}{2} + \dfrac{\sin(4x)}{8}$

② continued

Tatiana montoya
Section 2
5.7 trig integrals

④ $\int \cos^3(2x) = \underbrace{\cos^2(2x)}_{(1-\sin^2(2x))} \cos(2x)$

$(1-\sin^2(2x))\cos(2x)$

$u = \sin(2x)$

$du = 2\cos(2x)\,dx$

$\int \cos^3(2x)\,dx = \frac{1}{2}\int (1-u^2)\,du \rightarrow \frac{1}{2}\left(u - \frac{u^3}{3}\right)$

④ $\frac{1}{2}\sin(2x) - \frac{1}{6}\sin^3(2x)$

$\frac{1}{8}\left[\overset{①}{x} - \overset{②}{\frac{1}{2}\sin(2x)} - \overset{③}{\left(\frac{x}{2} + \frac{\sin(4x)}{8}\right)} + \overset{④}{\left(\frac{1}{2}\sin(2x) - \frac{1}{6}\sin^3(2x)\right)}\right]$

cancel cancel

$\frac{2x}{2} - \frac{x}{2} \rightarrow \frac{x}{2}$

$\frac{1}{8}\left[\frac{x}{2} - \frac{\sin(4x)}{8} - \frac{1}{6}\sin^3(2x)\right]$

$$\boxed{\frac{x}{16} - \frac{\sin(4x)}{64} - \frac{\sin^3(2x)}{48} + C}$$

Jiselle Haylie Lozano

# 5.9 (Simpson's Rule)

## Problem 1

Use Simpson's Rule with $n=4$ to approximate:

$$\int_0^2 (x^2+1)\,dx$$

Solution:

$$\Delta x = \frac{2-0}{4} = 0.5$$

Values:

$x_0 = 0,\ x_1 = 0.5,\ x_2 = 1,\ x_3 = 1.5,\ x_4 = 2$

$f(x) = x^2+1$ $\quad f(0.5) = 1.25$

$f(0) = 1$ $\quad f(1) = 2$

$f(1.5) = 3.25$ $\quad f(2) = 5$

Simpson's Rule:

$$S_4 = \frac{0.5}{3}[1+4(1.25)+2(2)+4(3.25)+5]$$

$$S_4 = \frac{0.5}{3}[1+5+4+13+5]$$

$$S_4 = \frac{0.5}{3}(28)$$

$$S_4 = 4.6667$$

Answer: $\boxed{4.667}$

Jiselle Haylie Lozano

# 5.9 (Simpson's Rule)

## Problem 2

Use Simpson's Rule with $n=6$ to approximate:

$$\int_1^4 \ln(x)dx$$

Solution:

$$\Delta x = \frac{4-1}{6} = 0.5$$

Values:

$x_0 = 1,\ x_1 = 1.5,\ x_2 = 2,\ x_3 = 2.5,\ x_4 = 3,\ x_5 = 3.5,\ x_6 = 4$

$f(x) = \ln(x)$

$f(1) = 0$

$f(1.5) = 0.4055$

$f(2) = 0.6931$

$f(2.5) = 0.9163$

$f(3) = 1.0986$

$f(3.5) = 1.2528$

$f(4) = 1.3863$

Simpson's Rule:

$$S_6 = \frac{0.5}{3}[0+4(0.4055)+2(0.6931)+4(0.9163)+2(1.0986)+4(1.2528)+1.3863]$$

$$S_6 = \frac{0.5}{3}[0+1.622+1.3862+3.6652+2.1972+5.0112+1.3863]$$

$$S_6 = \frac{0.5}{3}(15.2671)$$

$$S_6 = 2.5445$$

Answer:

$\boxed{2.545}$

5.9 (simpson)

1. $h = \frac{2-0}{4} = 0.5$

$x = 0, 0.5, 1, 1.5, 2$

$f(x) = x^2 + 1$

$f(0) = 1$
$f(0.5) = 1.25$
$f(1) = 2$
$f(1.5) = 3.25$
$f(2) = 5$

$= \frac{0.5}{3}\left[1 + 4(1.25) + 2(2) + 4(3.25) + 5\right]$

$\boxed{\approx 4.667}$

2. $h = \frac{\pi - 0}{6} = \frac{\pi}{6}$

$x = 0, \pi/6, 2\pi/6, 3\pi/6, 4\pi/6, 5\pi/6, \pi$

$f(0) = 0$

$f(\pi/6) = 0.271$

$f(\pi/3) = 0.889$

$f(\pi/2) = 0.623$

$f(2\pi/3) = -0.946$

$f(5\pi/6) = 0.545$

$f(\pi) = -0.430$

$= \frac{\pi/6}{3}\left[0 + 4(0.271) + 2(0.889) + 4(0.623) + 2(-0.946) + 4(0.545) + (-0.430)\right]$

$\boxed{= 0.910}$

# 5.9 Trap and Midtrap answers

**Problem # 1**

1. Approximate using
   a) trapezoidal rule $n=5$
   b) midpoint rule $n=5$
   c) underestimation/overestimation

$$\int_1^5 \frac{1}{x}\,dx \quad , n=5$$

① $\Delta x = \frac{b-a}{n} = \frac{5-1}{5} = 0.8$

② Find partition points

$x_0 = 1$
$x_1 = 1+0.8 = 1.08$
$x_2 = 1.08+0.8 = 2.6$
$x_3 = 2.6+0.8 = 3.4$
$x_4 = 3.4+0.8 = 4.2$
$x_5 = 4.2+0.8 = 5$

③ Evalute function $f(x) = \frac{1}{x}$

$f(x_0) = f(1) = 1$
$f(x_1) = f(1.8) = \frac{1}{1.8} = 0.5556$
$f(x_2) = f(2.6) = \frac{1}{2.6} = 0.3846$
$f(x_3) = f(3.4) = 0.2941$
$f(x_4) = f(4.2) = 0.2381$
$f(x_5) = f(5) = 0.2$

**a)** Trapezoidal rule

$T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + 2f(x_4) + f(x_5)\right]$

$T_5 = \frac{0.8}{2}\left[1 + 2(0.5556) + 2(0.3846) + 2(0.2941) + 2(0.2381) + 0.2\right]$

$= 0.4[4.1448]$

$\boxed{T_5 \approx 1.658}$

**b)** Midpoint Rule

$m_1 = \frac{1+1.8}{2} = 1.4$

$m_2 = \frac{1.8+2.6}{2} = 2.2$

$m_3 = \frac{2.6+3.4}{2} = 3$

$m_4 = \frac{3.4+4.2}{2} = 3.8$

$m_5 = \frac{4.2+5}{2} = 4.6$

$f(1.4) = \frac{1}{1.4} = 0.7143$
$f(2.2) = 0.4545$
$f(3) = 0.3333$
$f(3.8) = 0.2632$
$f(4.6) = 0.2174$

$M_n = \Delta x \sum f(\text{midpoints})$

$M_5 = 0.8(0.7143 + 0.4545 + 0.3333 + 0.2632 + 0.2174)$

$= 0.8(1.927)$

$\boxed{M_5 \approx 1.586}$

**c)** Over/under

- $f(x) = 1/x$ is concave up
- trapezoidal $\rightarrow$ overestimation
- midpoint $\rightarrow$ underestimation

# 5.9 Trap and Midtrap answers

Problem #2

2. Approximate using
   a) trapezoidal rule $n=6$
   b) midpoint rule $n=6$
   c) underestimation / overestimation

$$\int_0^3 \sqrt{x+1}\,dx, \quad n=6$$

① $\Delta x = \frac{3-0}{6} = 0.5$

② partition points
$0, 0.5, 1, 1.5, 2, 2.5, 3$

③ evaluate function $f(x) = \sqrt{x+1}$

$f(0) = \sqrt{0+1} = 1$

$f(0.5) = \sqrt{0.5+1} = 1.2247$

$f(1) = \sqrt{2} = 1.4142$

$f(1.5) = \sqrt{2.5} = 1.5811$

$f(2) = \sqrt{3} = 1.7321$

$f(2.5) = \sqrt{3.5} = 1.8708$

$f(3) = \sqrt{4} = 2$

**a)** Trapezoidal Rule

$$T_6 = \frac{0.5}{2}\left[f_0 + 2f_1 + 2f_2 + 2f_3 + 2f_4 + 2f_5 + f_6\right]$$

$$T_6 = 0.25\left[1 + 2(1.2247 + 1.4142 + 1.5811 + 1.7321 + 1.8708) + 2\right]$$

$$T_6 = 0.25\left[18.6458\right]$$

$$\boxed{T_6 \approx 4.662}$$

**b)** Midpoint Rule

Midpoints
$0.25, 0.75, 1.25, 1.75, 2.25, 2.75$

Evaluate

$f(0.25) = 1.1180$

$f(0.75) = 1.3229$

$f(1.25) = 1.5$

$f(1.75) = 1.6583$

$f(2.25) = 1.8020$

$f(2.75) = 1.9365$

$$M_n = \Delta x \sum f(\text{midpoints})$$

$$M_6 = 0.5\,(1.1180 + 1.3229 + 1.5 + 1.6583 + 1.8020 + 1.9365)$$

$$= 0.5(9.3377)$$

$$\boxed{M_6 \approx 4.669}$$

**c)** Over/under

- Function is concave down
- trapezoidal $\rightarrow$ underestimate
- Midpoint $\rightarrow$ overestimate

Dearyl Gemeniano

5.10 Type 1 : Intervals (Infinite)

① $\int_{-\infty}^{0} \frac{z}{z^4+4}\, dz$

Let

$u = z^2$

$du = 2z\, dz$

$\frac{1}{2} du = z\, dz$

$= \frac{1}{2}\int \frac{1}{u^2+4}\, du$

$= \frac{1}{2}\left[\frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right)\right]$

$= \frac{1}{4}\tan^{-1}\left(\frac{z^2}{2}\right)\Big|_{-\infty}^{0}$

$= \left(\frac{1}{4}\tan^{-1}\left(\frac{(0)^2}{2}\right)\right) - \left(\frac{1}{4}\underbrace{\tan^{-1}\left(\frac{(-\infty)^2}{2}\right)}_{\frac{\pi}{2}}\right)$

$= 0 - \frac{1}{4}\left(\frac{\pi}{2}\right)$

$= -\frac{\pi}{8}$

CONVERGENT

Dearyl Gemeniano

5.10 Type 1 : Intervals (Infinite)

(2) $\int_e^\infty \frac{1}{x(\ln x)^2}\,dx$

$\ln(e) = 1$

Let $\qquad$ when $x = e$, $u = 1$

$u = \ln x$ $\qquad$ when $x \to \infty$, $u \to \infty$

$du = \frac{1}{x}\,dx$

$$\int_1^\infty \frac{1}{u^2}\,du = \int_1^\infty u^{-2}\,du$$

$$= \left[-\frac{1}{u}\right]_1^\infty$$

$$= \left(-\frac{1}{\infty}\right) - \left(-\frac{1}{1}\right)$$

$$= 0 + 1$$

$$= 1$$

CONVERGENT

Kaija Holland

$\boxed{5.10}$ type 2 improper integrals

1) $\displaystyle\int_{-1}^{2} \frac{x}{(x+1)^2}\,dx$ convergent or divergent?

$= \lim_{t \to -1^+} \displaystyle\int_{t}^{2} \frac{x}{(x+1)^2}\,dx$

integration by parts

$\int u\,dv = uv - \int v\,du$

$$\begin{cases} u = x \\ dv = \dfrac{1}{(x+1)^2}\,dx \\ du = dx \\ v = \displaystyle\int \frac{1}{(x+1)^2}\,dx = -\frac{1}{x+1} \end{cases}$$

$= x\left(-\frac{1}{x+1}\right) - \int\left(-\frac{1}{x+1}\right)dx$

$= -\dfrac{x}{x+1} + \displaystyle\int \frac{1}{x+1}\,dx = \frac{-x}{x+1} + \ln|x+1| + C$

evaluate limit

$\lim_{t \to -1^+} \left[-\dfrac{x}{x+1} + \ln|x+1|\right]_t^2$

$x = 2$

$\dfrac{-2}{2+1} + \ln|2+1| = -\dfrac{2}{3} + \ln(3)$

$x = t$

$\dfrac{-t}{t+1} + \ln|t+1| = \dfrac{1}{0} = \infty$

$= \lim_{t \to -1^+} \left(-\dfrac{2}{3} + \ln(3) + \dfrac{t}{t+1} - \ln|t+1|\right)$

$= -\dfrac{2}{3} + \ln(3) + (-\infty) - (-\infty)$

$= -\dfrac{2}{3} + \ln(3) + \infty = \infty$

$\boxed{\text{DIVERGES}}$

Kaija Holland

$\boxed{5.10}$ type 2 improper integrals

2) $\int_0^{\pi} \frac{\sin^2 x}{\sqrt{x}}\,dx$ convergent or divergent? Use comparison theorem.

$0 \le f(x) \le g(x)$ for $(0, \pi)$

$0 \le \sin^2 x \le 1$ for all $x$

$0 \le \frac{\sin^2 x}{\sqrt{x}} \le \frac{1}{\sqrt{x}}$ $\to g(x)$

$g(x) = \frac{1}{\sqrt{x}} = x^{-1/2}$

p-series

$p = \frac{1}{2} < 1$ converges

Comparison Theorem

if $\int_a^b g(x)\,dx$ converges, then $\int_a^b f(x)\,dx$ also converges

when $0 \le f(x) \le g(x)$

Since $\int_0^{\pi} \frac{1}{\sqrt{x}}\,dx$ converges, $\int_0^{\pi} \frac{\sin^2 x}{\sqrt{x}}\,dx$ also $\boxed{\text{CONVERGES}}$

5.10 ( type II) : Michael Arellano

Determine whether the integral is convergent or divergent. Evaluate those that are convergent.

1) $\int_0^1 \frac{1}{x}\,dx$

→ the intergral is improper because at the lower limit as $x \to 0$ $\frac{1}{x} \to \infty$ ∴

$$\int_0^1 \frac{1}{x}\,dx$$

$$= \lim_{t\to 0} \int_t^1 \frac{1}{x}\,dx$$

$$= \lim_{t\to 0} \ln|x| \Big|_t^1$$

$$= \lim_{t\to 0} \ln|1| - \ln(t)$$

$$= 0 - (-\infty)$$

$$= \infty$$

⟶ Diverges

2) $\int_0^9 \frac{1}{\sqrt[3]{x-1}}\,dx$

→ the intergral is improper because at the lower limit as $x \to 1$ $\sqrt[3]{x-1} \to 0$ & $\frac{1}{0}$ ∴

$$\int_0^9 \frac{1}{\sqrt[3]{x-1}}\,dx$$

$$= \int_0^1 \frac{1}{\sqrt[3]{x-1}}\,dx + \int_1^9 \frac{1}{\sqrt[3]{x-1}}\,dx$$

$$= \int_0^1 (x-1)^{-1/3}\,dx + \int_1^9 (x-1)^{-1/3}\,dx$$

$$= \lim_{n\to 1^-} \int_0^t (x-1)^{-1/3}\,dx + \lim_{n\to 1^+} \int_0^t (x-1)^{-1/3}\,dx$$

$$= \lim_{n\to 1^-} \frac{(x-1)^{(-1/3+1)}}{(-1/3+1)}\Bigg|_0^t + \lim_{n\to 1^+} \frac{(x-1)^{(-1/3+1)}}{(-1/3+1)}\Bigg|_t^9$$

$$= \lim_{n\to 1^-} \frac{3}{2}(x-1)^{2/3}\Bigg|_0^t + \lim_{n\to 1^-} \frac{3}{2}(x-1)^{2/3}\Bigg|_t^9$$

$$= \left[\lim_{n\to 1^-} \frac{3}{2}(t-1)^{2/3} - \frac{3}{2}(0-1)^{2/3}\right] + \left[\lim_{n\to 1^-} \frac{3}{2}(9-1)^{2/3} - \lim_{n\to 1^-} \frac{3}{2}(t-1)^{2/3}\right]$$

$$= \left[\frac{3}{2}(1-1)^{2/3} - \frac{3}{2}(\ -1)^{2/3}\right] + \left[\frac{3}{2}(9-1)^{2/3} - \frac{3}{2}(1-1)^{2/3}\right]$$

$$= \left[-\frac{3}{2} - 0\right] + \left[\frac{3}{2}(8)^{2/3} - 0\right]$$

$$= \left[-\frac{3}{2} - 0\right] + \left[\frac{3}{2}(4)\right]$$

$$= -\frac{3}{2} + 6$$

$$\int_0^9 \frac{1}{\sqrt[3]{x-1}}\,dx = 9/2$$ — converges

# Type 1 Improper Integrals 5.10

Infinite Integrals

Tadhgan Harlin

## Solutions

1. $\int_{-\infty}^{\infty} x e^{-x^2} dx$

$$\int_{-\infty}^{\infty} x e^{-x^2} dx = \lim_{a \to -\infty} \int_a^0 x e^{-x^2} dx + \lim_{b \to \infty} \int_0^b x e^{-x^2} dx$$

$u = -x^2 \quad du = -2x\,dx \quad x\,dx = -\frac{1}{2} du$

$$\int_{-\infty}^{\infty} x e^{-x^2} dx = -\frac{1}{2}\int e^u du = -\frac{1}{2}e^u = -\frac{1}{2}e^{-x^2}$$

$$\lim_{a \to -\infty} \left[-\frac{1}{2}e^{-x^2}\right]_a^0 = \lim_{a \to -\infty} \left(-\frac{1}{2}e^0 - \left(-\frac{1}{2}e^{-a^2}\right)\right) = -\frac{1}{2} + 0 = -\frac{1}{2}$$

$$\lim_{b \to \infty} \left[-\frac{1}{2}e^{-x^2}\right]_b^0 = \lim_{b \to \infty} \left(-\frac{1}{2}e^{-b^2} - \left(-\frac{1}{2}e^0\right)\right) = 0 + \frac{1}{2} = \frac{1}{2}$$

$-\frac{1}{2} + \frac{1}{2} = 0$

$\boxed{\text{Converges to } 0}$

# Type 1 Improper Integrals 5.10
## Infinite Integrals
Tadhgan Harlin

## Solutions

2. $\int_0^\infty \sin\theta\, e^{\cos\theta}\, d\theta$

$\lim_{b\to\infty} \int_0^b \sin\theta\, e^{\cos\theta}\, d\theta$

$u = \cos\theta \quad du = -\sin\theta\, d\theta \quad \sin\theta\, d\theta = -du$

$\int \sin\theta\, e^{\cos\theta}\, d\theta = -\int e^u du = -e^u = -e^{\cos\theta}$

$\left[e^{\cos\theta}\right]_0^b = -e^{\cos b} - (-e^{\cos 0}) = -e^{\cos b} + e^1 = e - e^{\cos b}$

$e - e^{\cos\infty}$ $\boxed{\text{Diverges}}$

5.10 (Comp), Austin Berry, Math-151-03

55. $\int_0^\infty \frac{x}{x^3+1}\,dx$

$a_n = \int_0^\infty \frac{x}{x^3+1}\,dx \rightarrow \frac{1}{x^2+1}$

$b_n = \int_0^\infty \frac{1}{x^2}\,dx$

$x^2+1 > x^2 \rightarrow a_n < b_n$

$b_n$ converges by p-series test, $p=2>1$. Since $b_n > a_n$, $a_n$ converges by direct comparison test

60. $\int_0^\pi \frac{\sin^2 x}{\sqrt{x}}\,dx$

$a_n = \int_0^\pi \frac{\sin^2 x}{\sqrt{x}}\,dx$

$b_n = \int_0^\pi \frac{1}{\sqrt{x}}\,dx$

$a_n \leq b_n$

$\sin^2 x \leq 1$

$\int_0^\pi \frac{1}{\sqrt{x}}\,dx$

$\lim_{B\to 0^+} \int_B^\pi \frac{1}{\sqrt{x}}\,dx$

$2\sqrt{x}\Big|_B^\pi$

$\lim_{B\to 0^+} (2\sqrt{\pi} - 2\sqrt{B})$

$2\sqrt{\pi} - 0$

$2\sqrt{\pi}$

Converges ✓

Since $b_n$ converges and $a_n \leq b_n$ then $a_n$ also converges by direct comparison test

Gabriel Ballesteros

# Solutions 5.10 (Comp. Test)

57) $\int_1^\infty \frac{x+1}{\sqrt{x^4-x}}\,dx$

$= \frac{x+1}{(x^4-x)^{1/2}} = \frac{x+1}{x^2-x^{1/2}} = \frac{x}{x^2}$

$\frac{x+1}{\sqrt{x^4-x}} > \frac{x}{x^2}$

since $\frac{x}{x^2}$ diverges

$\frac{x+1}{\sqrt{x^4-x}}$ diverges

$\int_1^\infty \frac{x}{x^2} = \frac{1}{x}$

p-series $\leq 1$ so it diverges

58) $\int_0^\infty \frac{\arctan x}{2+e^x}\,dx$

$\arctan x < \frac{\pi}{2}$

$2+e^x > e^x$

$= \frac{\arctan x}{2+e^x} < \frac{\frac{\pi}{2}}{e^x}$

since $\frac{\frac{\pi}{2}}{e^x}$ converges

$\frac{\arctan x}{2+e^x}$ converges

$\lim_{t\to\infty} \int_0^t \frac{\pi}{2}e^{-x}\,dx$

$\lim_{t\to\infty} -\frac{\pi}{2}e^{-x}\Big|_0^t$

$\lim_{t\to\infty} -\frac{\pi}{2}e^{t} + \frac{\pi}{2}e^0$

$0 + \frac{\pi}{2}$

$= \frac{\pi}{2}$

converges