Alessandro

6 1

top line   bottom line

$\Downarrow$ $\Downarrow$

$$\int_0^1 e^x \, dx - \int_0^1 x e^{x^2}$$

$$e^x \Big|_0^1 - \frac{1}{2} e^{x^2} \Big|_0^1$$

$$\boxed{\frac{1}{2} e^1 - \frac{1}{2}}$$

Alessandro.

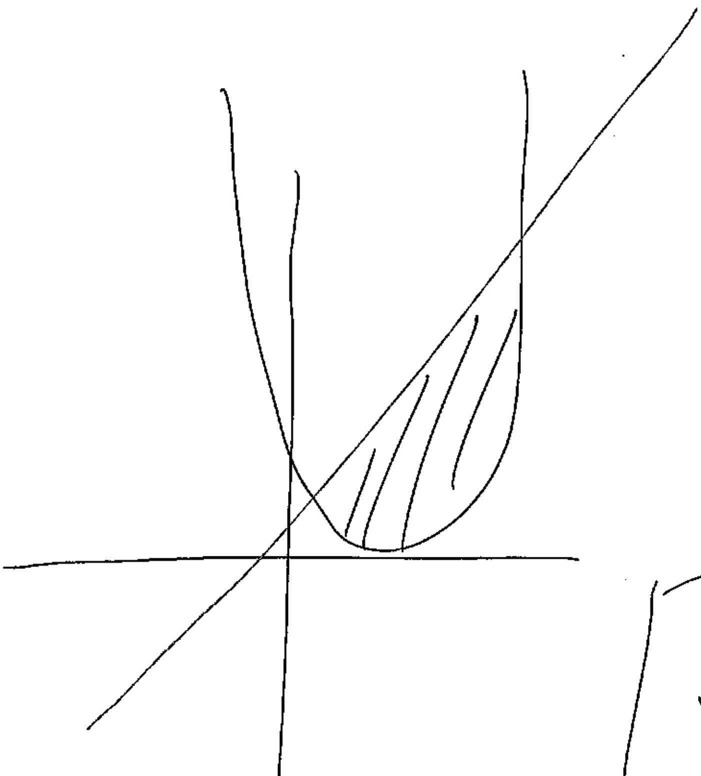
6.1

$$x^2 - 4x = 2x$$

$$x^2 - 6x = 0$$

$$x = 0 \qquad x = 6$$

where they intersect



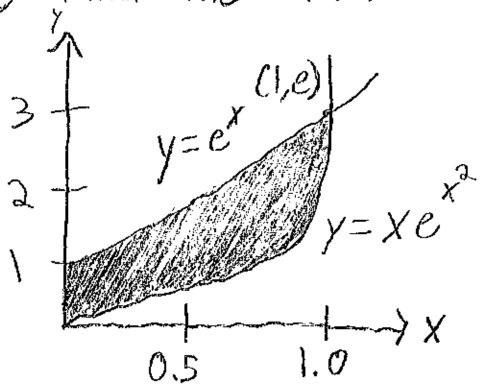
$$\int_0^6 2x\,dx - \int_0^6 x^2 - 4x$$

$$\boxed{x^2\Big|_0^6 - \left(\frac{x^3}{3} - 2x^2\right)\Big|_0^6}$$

Section 6.1 Solutions

Raina Pureth

(2) Find the area of the shaded region.



Area btwn. two curves: $\int_a^b f(x)-g(x)\,dx$

Upper curve: e^x

Lower curve: xe^{x^2}

Left bound: 0

Right bound: 1 (1,e)

Plug in

$$A=\int_0^1 (e^x - xe^{x^2})\,dx$$

Separate

$$A=\int_0^1 e^x dx - \int_0^1 xe^{x^2} dx$$

let $u=x^2$

$du=2x\,dx$

$\frac{1}{2}du = x\,dx$

$$e^x\Big|_0^1 = e^1 - e^0 = e-1$$

$$\frac{1}{2}\int_0^1 e^u\,du = \frac{1}{2}(e-1)$$

Solve

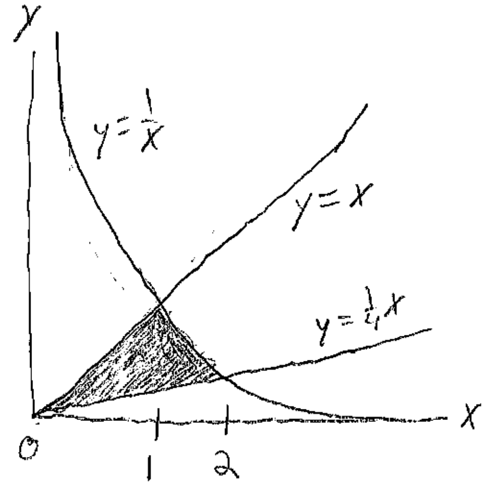
$$A=(e-1)-\frac{1}{2}(e-1) = \boxed{\frac{1}{2}(e-1)}$$

Section 6.1 Solutions

Rainn Puraih

(26) Sketch the regions enclosed by the graphs of the given functions and find the area of the region.

$y = \frac{1}{x}, \; y = x, \; y = \frac{1}{4}x, \; x > 0$



Identify intersection points:

- $y = x, \; y = \frac{1}{x}$
  - $x = \frac{1}{x} \Rightarrow x^2 = 1 \Rightarrow x = 1$
- $y = \frac{1}{4}x, \; y = \frac{1}{x}$
  - $\frac{1}{4}x = \frac{1}{x} \Rightarrow x^2 = 4 \Rightarrow x = 2$
- $y = x, \; y = \frac{1}{4}x$
  - $x = \frac{1}{4}x \Rightarrow x = 0$

Split into 2 regions

Region 1: from $x=0$ to $x=1$.
- $y = x$ Top
- $y = \frac{1}{4}x$ Bottom

$$\rightarrow \int_0^1 x - \frac{1}{4}x \, dx = \int_0^1 \frac{3}{4}x \, dx$$

$$= \frac{3}{4} \cdot \frac{x^2}{2}\Big|_0^1 = \frac{3}{8}x^2\Big|_0^1$$

$$= \frac{3}{8}(1)^2 - \frac{3}{8}(0)^2$$

$$= \boxed{\frac{3}{8}}$$

Region 2: from $x=1$ to $x=2$
- $y = \frac{1}{x}$ Top
- $y = \frac{1}{4}x$ Bottom

$$\rightarrow \int_1^2 \frac{1}{x} - \frac{1}{4}x \, dx$$

$$= \ln|x| - \frac{1}{8}x^2\Big|_1^2$$

$$= \ln|2| - \frac{1}{8}(4) - \left(\ln|1| - \frac{1}{8}(1)^2\right)$$

$$= \ln|2| - \frac{1}{2} + \frac{1}{8} = \boxed{\ln|2| - \frac{3}{8}}$$

$$-\frac{4}{8} + \frac{1}{8}$$

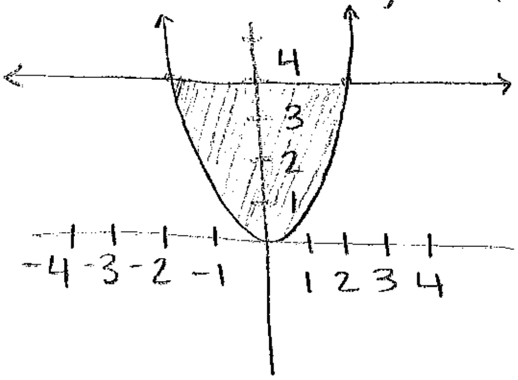
Total Area

A = Region 1 + Region 2

$$A = \frac{3}{8} + \ln|2| - \frac{3}{8} = \boxed{\ln|2|}$$

6.2 Volumes Audrey Hemiges

1. $y = x^2$ and $y = 4$; about the x-axis



$x^2 = 4$

$x = \sqrt{4} = \pm 2$ ← region is between -2 and 2

@ x-axis → washer

outer radius $R(x) = 4$

inner radius $r(x) = x^2$

Washer formula

$$V = \pi \int_a^b (R^2 - r^2)\,dx \quad \text{so} \quad V = \pi \int_{-2}^{2} (4^2 - (x^2)^2)\,dx$$

$$= \pi \int_{-2}^{2} (16 - x^4)\,dx$$

Integrate

$$\pi \left[\int_{-2}^{2} 16\,dx - \int_{-2}^{2} x^4\,dx \right]$$

$$\int_{-2}^{2} 16\,dx = 16x \Big|_{-2}^{2} = 16(2) - 16(-2) = 64$$

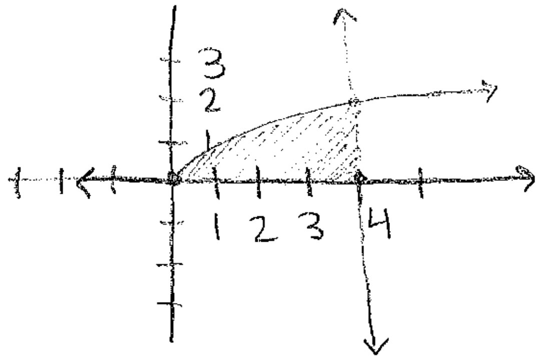
$$\int_{-2}^{2} x^4\,dx = 2\int_0^2 x^4\,dx = 2\left[\frac{x^5}{5}\right]_0^2 = 2 \cdot \frac{32}{5} = \frac{64}{5}$$

$$V = \pi\left(64 - \frac{64}{5}\right) = \pi \cdot \frac{256}{5}$$

$$\boxed{V = \frac{256\pi}{5}}$$

6.2 Volumes Audrey Herriges

2. $y=\sqrt{x}$, $y=0$, $x=4$; about the x-axis



left endpoint : $x=0$
right endpoint : $x=4$
$0 \le x \le 4$

@ x-axis → disk since $y=0$
$R(x)=\sqrt{x}$

Disk formula

$V=\pi\int_a^b [R(x)]^2\,dx$ so $V=\pi\int_0^4 (\sqrt{x})^2\,dx$

Integrate

$\pi\int_0^4 x\,dx = \left[\frac{x^2}{2}\right]_0^4 = \frac{16}{2} = 8$

$\boxed{V=8\pi}$

Briana Carrasco
section 8

H.1
Curves in Polar coordinates

Problem 1

- To plot $(3, \pi/4)$
  1. find angle $\pi/4$ (45°) on the grid
  2. Move 3 units out from the pole

- Finding other coordinates
  - With $r>0$: add or subtract a full rotation (2π)

$$\theta = \frac{\pi}{4} - 2\pi = -\frac{7\pi}{4}$$

Pair 1: $(3, -\frac{7\pi}{4})$

  - With $r<0$: find the opposite angle $(\pi/4 + \pi)$ and make radius negat

$$\theta = \frac{\pi}{4} + \pi = \frac{5\pi}{4}$$

Pair 2: $(-3, \frac{5\pi}{4})$

Problem 2

- To convert $(2, 5\pi/6)$ to cartesian (x, y), use:
  - $x = r\cos(\theta)$
  - $y = r\sin(\theta)$

1) substitute given values

$x = 2\cos(5\pi/6)$
$y = 2\sin(5\pi/6)$

2) unit circle for $5\pi/6 = 150°$
- $\cos(5\pi/6) = -\sqrt{3}/2$
- $\sin(5\pi/6) = 1/2$

3) solve

$x = 2(-\sqrt{3}/2) = -\sqrt{3}$
$y = 2(1/2) = 1$

Cartesian coordinates:
$(-\sqrt{3}, 1)$

6.4 KEY

1. $y = 1 + 6x^{3/2}$, $0 \leq x \leq 1$

$$\frac{dy}{dx} = \frac{d}{dx}(1 + 6x^{3/2})$$

$$\frac{dy}{dx} = 0 + 6 \cdot \frac{3}{2} x^{(3/2-1)}$$

$$\frac{dy}{dx} = 9x^{1/2}$$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\int_0^1 \sqrt{1 + 81x}\, dx$$

U-substitution

$u = 1 + 81x$ $\quad du = \frac{1}{81} dx$

when $x = 0$, $u = 1 + 81(0) = 1$

when $x = 1$, $u = 1 + 81(1) = 82$

$$\int_1^{82} \sqrt{u}\left(\frac{1}{81} du\right)$$

$$L = \frac{1}{81}\left[\frac{2}{3} u^{3/2}\right]_1^{82}$$

$$L = \frac{2}{243}\left(\frac{2}{3}(82)^{3/2} - \frac{2}{3}(1)^{3/2}\right)$$

$$\boxed{L = \frac{2}{243}\left((82)^{3/2} - 1\right)}$$

JOSLYN PHILSON

Thomas Cooper

6.4 Parametric Equations

6.4.35: Use either a CAS or the Table of Integrals to find the exact length of the curve:

$$x = t^3, \qquad y = t^4, \qquad 0 \le t \le 1$$

First, we use the arc length formula to find the length of the curve:

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

We'll derive x and y to find $\frac{dx}{dt}$ and $\frac{dy}{dt}$

$$x = t^3 \qquad y = t^4$$

$$\frac{dx}{dt} = 3t^2 \qquad \frac{dy}{dt} = 4t^3$$

Now we'll plug in our known values, along with our bounds:

$$L = \int_0^1 \sqrt{(3t^2)^2 + (4t^3)^2}\, dt$$

distribute the exponents

And some algebra to simplify:

$$L = \int_0^1 \sqrt{9t^4 + 16t^6}\, dt$$

factor out t^4

$$L = \int_0^1 \sqrt{t^4(9 + 16t^2)}\, dt$$

complete the root function on t^4 and pull it out of the root

$$L = \int_0^1 t^2\sqrt{9 + (4t)^2}\, dt$$

Thomas Cooper

This integral fits the form $\sqrt{a^2+x^2}$, signaling a tangent trig sub:

$$\sqrt{a^2+x^2} \quad \rightarrow \quad \text{use } x = a\tan\theta$$

$$\sqrt{3^2+4t^2} \quad \rightarrow \quad 4t = 3\tan\theta$$

$$4t = 3\tan\theta \quad \rightarrow \quad t = \frac{3}{4}\tan\theta$$

$$t = \frac{3}{4}\tan\theta \quad \rightarrow \quad dt = \frac{3}{4}\sec^2\theta\, d\theta$$

Substitute the trig sub values we found:

$$\sqrt{9+(4t)^2} \quad \rightarrow \quad \sqrt{3^2+3^2\tan^2\theta} \quad \rightarrow \quad \sqrt{3^2(1+\tan^2\theta)} \quad \rightarrow \quad 3\sec\theta$$

$$t^2 \quad \rightarrow \quad \left(\frac{3}{4}\tan\theta\right)^2$$

$1+\tan^2\theta = \sec^2\theta$

$$dt \quad \rightarrow \quad \left(\frac{3}{4}\sec^2\theta\right)$$

Plug substituted trig values into the arc length formula:

$$L = \int_{t=0}^{t=1} \left(\frac{3}{4}\tan\theta\right)^2 (3\sec\theta)\left(\frac{3}{4}\sec^2\theta\right) d\theta$$

distribute exponents
pull out constants
combine secent functions

$$L = \frac{81}{64}\int_{t=0}^{t=1} \tan^2\theta\sec^3\theta\, d\theta$$

Use trig identity to convert $\tan^2\theta$ to $\sec^2\theta - 1$ and distribute:

$$L = \frac{81}{64}\int_{t=0}^{t=1} \sec^5\theta - \sec^3\theta\, d\theta$$

split into two integrals

$$L = \frac{81}{64}\left[\int_{t=0}^{t=1} \sec^5\theta\, d\theta - \int_{t=0}^{t=1} \sec^3\theta\, d\theta\right]$$

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Use a table of integrals or integration by parts to solve the $\sec^3\theta$ integral:

Remember the integration by parts formula: $\int u\,dv = uv - \int v\,du$

$$\int_{t=0}^{t=1} \sec^3\theta\,d\theta = \int_{t=0}^{t=1} \sec^2\theta\,\sec\theta\,d\theta$$

$$u = \sec\theta \qquad dv = \sec^2\theta$$

$$du = \sec\theta\,\tan\theta \qquad v = \tan\theta$$

Plugging into integration by parts formula:

$$\int_{t=0}^{t=1} \sec^3\theta\,d\theta = \sec\theta\,\tan\theta - \int_{t=0}^{t=1} \tan^2\theta\,\sec\theta\,d\theta$$

Use trig identity to convert $\tan^2\theta$ to $\sec^2\theta - 1$ and distribute again:

$$\int_{t=0}^{t=1} \sec^3\theta\,d\theta = \sec\theta\,\tan\theta - \int_{t=0}^{t=1} (\sec^2\theta - 1)\sec\theta\,d\theta$$

$$\int_{t=0}^{t=1} \sec^3\theta\,d\theta = \sec\theta\,\tan\theta - \int_{t=0}^{t=1} \sec^3\theta - \sec\theta\,d\theta$$

split into two integrals

$$\boxed{\int_{t=0}^{t=1} \sec^3\theta\,d\theta} = \sec\theta\,\tan\theta - \boxed{\int_{t=0}^{t=1} \sec^3\theta\,d\theta} + \int_{t=0}^{t=1} \sec\theta\,d\theta$$

Use algebra to combine the 2 same integrals to one side of the equation:

$$\boxed{2\int_{t=0}^{t=1} \sec^3\theta\,d\theta = \sec\theta\,\tan\theta + \int_{t=0}^{t=1} \sec\theta\,d\theta}$$

**

Thomas Cooper

Use table of integrals or algebra and u substitution to solve the integral of sec θ

$$\int_{t=0}^{t=1} \sec\theta \, d\theta = \int_{t=0}^{t=1} \sec\theta \, \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta} \, d\theta = \int_{t=0}^{t=1} \frac{\sec^2\theta + \sec\theta \, \tan\theta}{\sec\theta + \tan\theta} \, d\theta$$

$$u = \sec\theta + \tan\theta$$

$$du = \sec^2\theta + \sec\theta \, \tan\theta$$

$$\int_{t=0}^{t=1} \frac{1}{u} \, du = [\ln|u|]_{t=0}^{t=1}$$

$$\int_{t=0}^{t=1} \sec\theta \, d\theta = [\ln|\sec\theta + \tan\theta|]_{t=0}^{t=1}$$

**plug back into equation found on last page

$$\int_{t=0}^{t=1} \sec^3\theta \, d\theta = \frac{\sec\theta \, \tan\theta + [\ln|\sec\theta + \tan\theta|]_{t=0}^{t=1}}{2}$$

Do the same with the $\sec^5\theta$ integral:

$$\int_{t=0}^{t=1} \sec^5\theta \, d\theta = \int_{t=0}^{t=1} \sec^3\theta \sec^2\theta \, d\theta$$

$$u = \sec^3\theta \quad dv = \sec^2\theta$$

$$du = 3\sec^2\theta \sec\theta \, \tan\theta \quad v = \tan\theta$$

$$\int_{t=0}^{t=1} \sec^5\theta \, d\theta = \sec^3\theta \, \tan\theta - \int_{t=0}^{t=1} 3\sec^3\theta \tan^2\theta \, d\theta$$

$$\int_{t=0}^{t=1} \sec^5\theta \, d\theta = \sec^3\theta \, \tan\theta - 3\int_{t=0}^{t=1} \sec^3\theta \, (\sec^2\theta - 1) \, d\theta$$

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$$\int_{t=0}^{t=1} \sec^5\theta\, d\theta = \sec^3\theta\,\tan\theta - 3\int_{t=0}^{t=1} \sec^5\theta - \sec^3\theta\, d\theta$$

$$\int_{t=0}^{t=1} \sec^5\theta\, d\theta = \sec^3\theta\,\tan\theta - 3\int_{t=0}^{t=1} \sec^5\theta + 3\int_{t=0}^{t=1} \sec^3\theta\, d\theta$$

$$4\int_{t=0}^{t=1} \sec^5\theta\, d\theta = \sec^3\theta\,\tan\theta + 3\int_{t=0}^{t=1} \sec^3\theta\, d\theta$$

*notice we found the integral of $\sec^3\theta\, d\theta$ on page 4

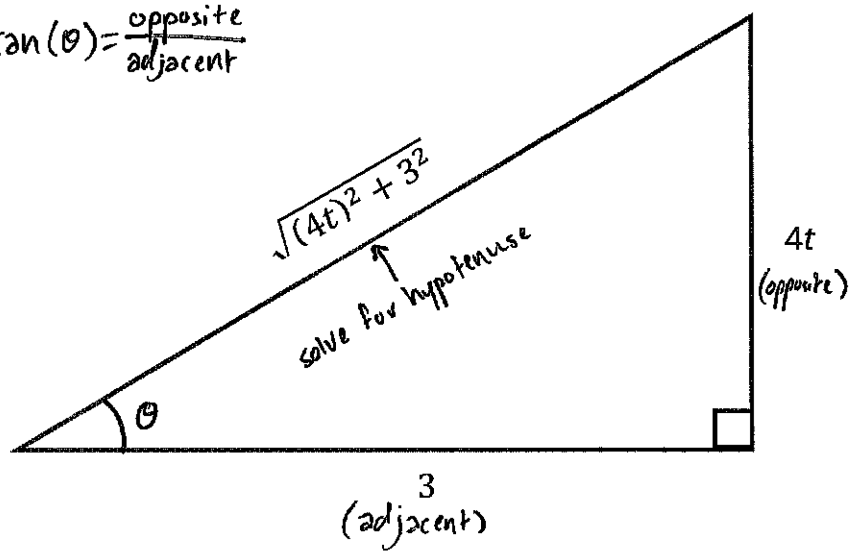
$$4\int_{t=0}^{t=1} \sec^5\theta\, d\theta = \sec^3\theta\,\tan\theta + 3\left[\frac{\sec\theta\,\tan\theta + [\ln|\sec\theta + \tan\theta|]_{t=0}^{t=1}}{2}\right]$$

$$\int_{t=0}^{t=1} \sec^5\theta\, d\theta = \frac{\sec^3\theta\,\tan\theta + 3\left[\frac{\sec\theta\,\tan\theta + [\ln|\sec\theta + \tan\theta|]_{t=0}^{t=1}}{2}\right]}{4}$$

Finally, we replace our trig sub values back to their original t values and plug them all back into the arc length formula:

$\tan\theta = \frac{4t}{3}$ *with this we can create a right triangle to find secant values

$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$



$$\sec\theta = \frac{hypotenuse}{adjacent}$$

$$\sec\theta = \frac{\sqrt{(4t)^2 + 3^2}}{3}$$

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$$L = \frac{81}{64}\left[\int_{t=0}^{t=1} \sec^5\theta\, d\theta - \int_{t=0}^{t=1} \sec^3\theta\, d\theta\right]$$

$$\int_{t=0}^{t=1} \sec^5\theta\, d\theta = \frac{\sec^3\theta\ \tan\theta + 3\left[\frac{\sec\theta\ \tan\theta\ + [\ln|\sec\theta + \tan\theta|]_{t=0}^{t=1}}{2}\right]}{4}$$

$$\int_{t=0}^{t=1} \sec^5\theta\, d\theta = \frac{\sec^3\theta\ \tan\theta}{4} + \frac{3(\sec\theta\ \tan\theta)}{8} + \frac{3[\ln|\sec\theta + \tan\theta|]_{t=0}^{t=1}}{8}$$

$$\int_{t=0}^{t=1} \sec^3\theta\, d\theta = \frac{\sec\theta\ \tan\theta\ + [\ln|\sec\theta + \tan\theta|]_{t=0}^{t=1}}{2}$$

$$\int_{t=0}^{t=1} \sec^3\theta\, d\theta = \frac{\sec\theta\ \tan\theta}{2} + \frac{[\ln|\sec\theta + \tan\theta|]_{t=0}^{t=1}}{2}$$

$$\boxed{\int_{t=0}^{t=1} \sec^5\theta\, d\theta} - \boxed{\int_{t=0}^{t=1} \sec^3\theta\, d\theta} =$$

$$\frac{\sec^3\theta\ \tan\theta}{4} + \frac{3(\sec\theta\ \tan\theta)}{8} + \frac{3[\ln|\sec\theta + \tan\theta|]_{t=0}^{t=1}}{8} - \frac{\sec\theta\ \tan\theta}{2} - \frac{[\ln|\sec\theta + \tan\theta|]_{t=0}^{t=1}}{2}$$

*combine like terms

$$\int_{t=0}^{t=1} \sec^5\theta\, d\theta - \int_{t=0}^{t=1} \sec^3\theta\, d\theta = \frac{\sec^3\theta\ \tan\theta}{4} - \frac{(\sec\theta\ \tan\theta)}{8} - \frac{[\ln|\sec\theta + \tan\theta|]_{t=0}^{t=1}}{8}$$

Substitute t values back into the equation:

$$\tan\theta = \frac{4t}{3} \qquad \sec\theta = \frac{\sqrt{(4t)^2 + 3^2}}{3}$$

$$\int_{t=0}^{t=1} \sec^5\theta\, d\theta - \int_{t=0}^{t=1} \sec^3\theta\, d\theta = \frac{\left(\frac{\sqrt{(4t)^2+3^2}}{3}\right)^3\left(\frac{4t}{3}\right)}{4} - \frac{\left(\frac{\sqrt{(4t)^2+3^2}}{3}\right)\left(\frac{4t}{3}\right)}{8} - \frac{\left[\ln\left|\left(\frac{\sqrt{(4t)^2+3^2}}{3}\right) + \left(\frac{4t}{3}\right)\right|\right]_0^1}{8}$$

$$\int_{t=0}^{t=1} \sec^5\theta\, d\theta - \int_{t=0}^{t=1} \sec^3\theta\, d\theta = \frac{t((4t)^2+3^2)\sqrt{(4t)^2+3^2}}{81} - \frac{t\sqrt{(4t)^2+3^2}}{18} - \frac{1}{8}\left[\ln\left|\frac{\sqrt{(4t)^2+3^2} + 4t}{3}\right|\right]_0^1$$

$$\int_{t=0}^{t=1} \sec^5\theta\, d\theta - \int_{t=0}^{t=1} \sec^3\theta\, d\theta = \frac{t((4t)^2+3^2)\sqrt{(4t)^2+3^2}}{81} - \frac{t\sqrt{(4t)^2+3^2}}{18} - \frac{1}{8}\ln\left|\frac{\sqrt{(4t)^2+3^2} + 4t}{3}\right|$$

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Plug into the arc length formula:

$$L = \left[\frac{81}{64}\left[\frac{t((4t)^2+3^2)\sqrt{(4t)^2+3^2}}{81} - \frac{t\sqrt{(4t)^2+3^2}}{18} - \frac{1}{8}\ln\left|\frac{\sqrt{(4t)^2+3^2}+4t}{3}\right|\right]\right]_0^1$$

$$L = \left[\frac{t((4t)^2+3^2)\sqrt{(4t)^2+3^2}}{64} - \frac{9t\sqrt{(4t)^2+3^2}}{128} - \frac{81}{512}\ln\left|\frac{\sqrt{(4t)^2+3^2}+4t}{3}\right|\right]_0^1$$

Evaluated from 0 to 1…

$$L(1) = \frac{(4^2+3^2)\sqrt{4^2+3^2}}{64} - \frac{9\sqrt{4^2+3^2}}{128} - \frac{81}{512}\ln\left|\frac{\sqrt{4^2+3^2}+4}{3}\right|$$

$$L(1) = \frac{(25)\sqrt{25}}{64} - \frac{9\sqrt{25}}{128} - \frac{81}{512}\ln\left|\frac{\sqrt{25}+4}{3}\right|$$

$$L(1) = \frac{125}{64} - \frac{45}{128} - \frac{81}{512}\ln\left|\frac{9}{3}\right|$$

$$L(1) = \frac{205}{128} - \frac{81}{512}\ln|3|$$

$$L(0) = \frac{0(0^2+3^2)\sqrt{0^2+3^2}}{64} - \frac{0\sqrt{0^2+3^2}}{128} - \frac{81}{512}\ln\left|\frac{\sqrt{0^2+3^2}+0}{3}\right|$$

$$L(0) = 0 - \frac{81}{512}\ln\left|\frac{3}{3}\right|$$

$$L(0) = -\frac{81}{512}\ln|1|$$

$$L(0) = 0$$

$$L(1) - L(0) = \frac{205}{128} - \frac{81}{512}\ln|3|$$

$$Arc\ Length\ (L) = \frac{205}{128} - \frac{81}{512}\ln|3| \approx 1.42775860277$$

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6.4 Parametric Equations

6.4.21: Find the exact length of the curve:

$$x = 1 + 3t^2, \qquad y = 4 + 2t^3, \qquad 0 \leq t \leq 1$$

First, we use the arc length formula to find the length of the curve:

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

We'll derive x and y to find $\frac{dx}{dt}$ and $\frac{dy}{dt}$

$$x = 1 + 3t^2 \qquad y = 4 + 2t^3$$

$$\frac{dx}{dt} = 6t \qquad \frac{dy}{dt} = 6t^2$$

Now we'll plug in our known values, along with our bounds:

$$L = \int_0^1 \sqrt{(6t)^2 + (6t^2)^2}\, dt$$

And some algebra to simplify:

$$L = \int_0^1 \sqrt{36t^2 + 36t^4}\, dt$$

$$L = \int_0^1 \sqrt{36t^2(1 + t^2)}\, dt$$

Thomas Cooper

$$L = \int_0^1 6t\sqrt{1+t^2}\,dt$$

Now we solve using u-substitution:

$$u = 1+t^2$$

$$du = 2tdt$$

$$\int_{t=0}^{t=1} 3\sqrt{u} = 3\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_{t=0}^{t=1}$$

$$3\left[\frac{2u^{\frac{3}{2}}}{3}\right]_{t=0}^{t=1} = 2\left[u^{\frac{3}{2}}\right]_{t=0}^{t=1} = 2\left[(1+t^2)^{\frac{3}{2}}\right]_0^1$$

$$2\left[\left[(1+1^2)^{\frac{3}{2}}\right]-\left[(1+0^2)^{\frac{3}{2}}\right]\right] = 2\left[2\sqrt{2}-1\sqrt{1}\right] = 4\sqrt{2}-2$$

6.6

Find moments and center of mass of system of objects that have masses 3, 4, 8 points $(-1,1)$, $(2,-1)$ and $(3,2)$

13. $x = t\cos t,\ y = t\sin t,\quad 0 \le t \le 2\pi$

$\frac{dx}{dt} = \cos t - t\sin t$

$\frac{dy}{dt} = \sin t + t\cos t$

$L = \int_0^{2\pi} \sqrt{(\cos t - t\sin t)^2 + (\sin t + t\cos t)^2}\,dt$

$= \cos^2 t - 2t\sin t\cos t + t^2\sin^2 t$

$= \sin^2 t + 2t\cos t + t^2$

$\cos^2 t + \sin^2 t + t^2(\sin^2 t + \cos^2 t)$

$= 1 + t^2$

So, $L = \int_0^{2\pi} \sqrt{1+t^2}\,dt$

≈ 21.256

12. $x = t + \cos t,\ y = t - \sin t,\quad 0 \le t \le 2\pi$

$\frac{dx}{dt} = 1 - \sin t$

$\frac{dy}{dt} = 1 - \cos t$

$L = \int_0^{2\pi} \sqrt{(1-\sin t)^2 + (1-\cos t)^2}\,dt$

$= 1 - 2\sin t + \sin^2 t$ $\qquad$ $= 1 - 2\cos t + \cos^2 t$

$2 - 2\sin t - 2\cos t + \sin^2 t + \cos^2 t$

$\sin^2 t + \cos^2 t = 1$ So, $= 3 - 2\sin t - 2\cos t$

$L = \int_0^{2\pi} \sqrt{3 - 2\sin t - 2\cos t}\,dt$

≈ 11.811

Nicolas Guillen | 6.4 Parametric

Jovan Contreras
Math 151-02

Solutions

Section 6.4

Solution to promblem 1:

Step 1: Use Hooke's Law.

Hooke's Law states that the force f needed to strecth a spring x units beyond its natural length is $F(x)=Kx$, where K is the spring constant.

Step 2: Find the spring constant (K)

the natural length is 15 cm and it is strected to 20 cm

the distance strected is $x=20-15=5\text{cm}$

Converting to meters: $x=0.05\text{ m}$

$$30=K(0.05)$$

$$K=\frac{30}{0.05}=600\text{N/m}$$

Step 3: Set up the work integral.

we want to strecth the spring from 15 cm (where $x=0$) to 25 cm (where $x=25-15=10\text{cm}$, or 0.10m).

$$W=\int_0^{0.10} 600x\,dx$$

Step 4: Solve the integral.

$$W=\left[300x^2\right]_0^{0.10}$$

$$W=300(0.10)^2-300(0)^2=300(0.01)=3\text{ Joules}$$

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Math 151-02

Solution to promblem 2: Section 6.4

Step 1: understand the variable force.

As we pull the cable up, the part hanging over the side gets shorter and lighter. This means we are doing less work as we go. we need to find the work required to lift a tiny "slice" of the cable

Step 2: Set up the coordinate system

Let x be the distance from the top of the building, we look at a small segment of the cable with length Δx at a distance x from the top.

Step 3: Find the weight (force) of the segment

The mass of the segment is Density × Length $= 3\Delta x$

The force (weight) is mass × Gravity $= (3\Delta x)(9.8) = 29.4\Delta x$

Step 4: Determine the distance moved

the segment at distance x must be pulled up to the top so it travels a distance of x.

Step 5: Integrate over the length of the cable.

The cable is 20 meters long, so we integrate from $x=0$ to $x=20$.

$$W = \int_0^{20} 29.4x\,dx$$

Step 6: Evaluate the integral

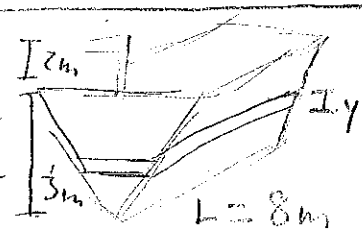
$$W = \left[\frac{29.4}{2}x^2\right]_0^{20} = \left[14.7x^2\right]_0^{20}$$

$$W = 14.7(20)^2 - 14.7(0)^2$$

$$W = 5{,}880 \text{ Joules}$$

Maesan Harris Sec. 6.6 Tan

1.) Density $= 62.5 \text{ lb/ft}^3 = 9800 \text{ N/m}^3$



Area of slice $= 8 \cdot y$

Volume $=$ Area dy $= 8y\,dy$ → represents change in y variable

Distance to top $= (3-y)+2 = (5-y)$

$$W = \int_{\text{Bottom of Tank}}^{\text{Top of Tank}} (\text{density})(\text{distance})(\text{area})\,dy$$

$$W = \int_0^3 (9800)(5-y)(8y)\,dy$$

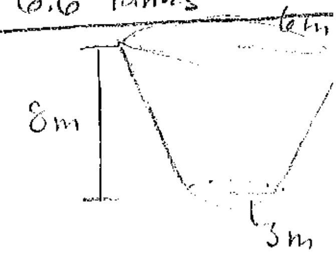
$$= 78{,}400 \int_0^3 (5-y)y\,dy$$

$$= 78{,}400 \int_0^3 (5y-y^2)\,dy$$

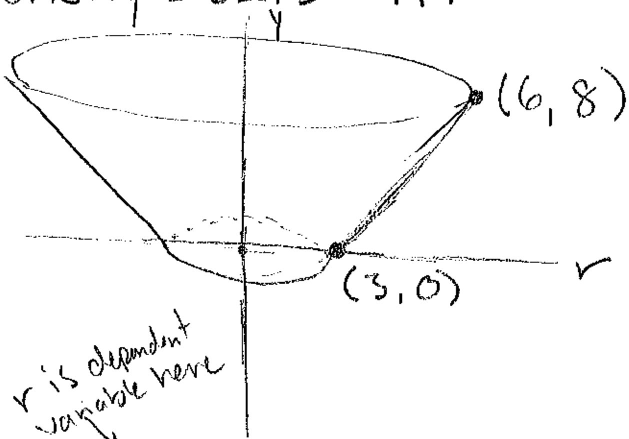
$$= 78{,}400\left[\left(\tfrac{5}{2}y^2 - \tfrac{1}{3}y^3\right)\Big|_0^3\right]$$

$$= 78{,}400\left(\tfrac{5}{2}(3^2) - \tfrac{1}{3}(3)^3\right) \boxed{= 1{,}058{,}400 \text{ J}}$$

Maeson Harris Sec. 6.6 Tanks



2.) Density = 62.5 lb/ft^3



r is dependent variable here

$$\text{Slope} = \frac{\Delta r}{\Delta y} = \frac{6-3}{8-0} = \frac{3}{8} = 0.375$$

r intercept

$$r(y) = 3 + 0.375y$$

Area of slice: $A = \pi r^2 = \pi (r(y))^2 = \pi (3 + 0.375y)^2$

Volume of slice: $V = \pi (3+0.375y)^2 dy$

Distance to top: $(8-y)$

$$W = \int_{\text{Bottom}}^{\text{Top}} (\text{Density})(\text{Area})(\text{Distance})\, dy$$

$$W = \int_0^8 (62.5)(\pi)(3+0.375y)^2(8-y)\, dy$$

$$= 62.5\pi \int_0^8 (3+0.375y)^2 (8-y)\, dy$$

$$= 62.5\pi \int_0^8 (72 + 9y - 1.125y^2 - 0.140625y^3)\, dy$$

$$= 62.5\pi \left[\left(72y + \frac{9}{2}y^2 - \frac{1.125}{3}y^3 - \frac{0.140625}{4}y^4 \right) \Big|_0^8 \right)$$

$$= 62.5\pi\,(576 + 288 - 192 - 144)$$

$$\boxed{= 33{,}000\pi \text{ ft-lb} = 1.04 \times 10^5 \text{ ft-lb}}$$

Lorena Torres Gonzalez Section 6.6 (emptying tanks)

(1) A 10-kg bucket is lifted from the ground to a height of 12 m at a constant speed with a rope that weighs 0.8 kg/m. Initially, the bucket contains 36 kg of water. However, water leaks from the bucket at a constant rate beginning when the bucket is lifted and finishes draining just as the bucket reaches the 12 m level. How much work is done?

Given : $m_{bucket} = 10\text{ kg}$; $\rho_{rope} = 0.8\text{ kg/m}$; $h = 12\text{ m}$

$m_{i\,water} = 36\text{ kg}$

$$F_{bucket} = 10g$$

$$F_{rope} = 0.8(12-x)g$$

$$F_{water} = 36\left(1 - \frac{x}{12}\right)g$$

$$= (36 - 3x)g$$

Total Force :

$$F(x) = g[10 + 0.8(12-x) + (36-3x)]$$

$$= g[10 + 9.6 - 0.8x + 36 - 3x]$$

$$= g[55.6 - 3.8x]$$

Work integral :

$$W = \int_0^{12} g(55.6 - 3.8x)\,dx$$

$$= 9.8\left[55.6x - 1.9x^2\right]_0^{12}$$

$$= 9.8\left[55.6(12) - 1.9(12)^2\right]$$

$$= 9.8\left[667.2 - 273.6\right]$$

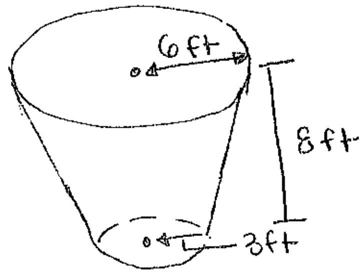
$$= 9.8(393.6)$$

$$\boxed{W = 3857.28\text{ J}}$$

Lorena Torres Gonzalez Section 6.6 (emptying tanks)

(2) A tank is full of water. Find the work required to pump the water out of the spout. Use the fact that water weighs 62.5 lb/ft^3

Frustum of a cone



Given: Top radius = 6ft ; Bottom radius = 3ft ; h = 8ft
water weight density = 62.5 lb/ft^3

$$r(x) = 3 + \frac{6-3}{8}x$$

$$r(x) = 3 + \frac{3x}{8}$$

Area of a horizontal slice

$$A(x) = \pi r(x)^2 = \pi\left(3 + \frac{3x}{8}\right)^2$$

$$dw = (62.5)\left[\pi\left(3 + \frac{3x}{8}\right)^2\right](8-x)\,dx$$

$$W = 62.5\pi \int_0^8 \left(3 + \frac{3x}{8}\right)^2 (8-x)\,dx$$

$$= 62.5\pi \int_0^8 \left(9 + \frac{9x}{4} + \frac{9x^2}{64}\right)(8-x)\,dx$$

$$= 62.5\pi \int_0^8 \left(72 - 9x + 18x - \frac{9x^2}{4} + \frac{9x^2}{8} - \frac{9x^3}{64}\right)dx$$

$$= 62.5\pi \int_0^8 \left(72 + 9x - \frac{9x^2}{8} - \frac{9x^3}{64}\right)dx$$

$$= 62.5\pi\,[576 + 288 - 192 - 144]$$

$$= 62.5\pi\,[528]$$

$$\boxed{W = 33000\pi \approx 1.04 \times 10^5 \text{ ft-lb}}$$

6.6 Angela Cha

9) a spring has a natural length of 20 cm. Compare the work W_1 Done in stretching the spring from 20 cm to 30 cm with the W_2 done in stretching it from 30 cm to 40 cm. How are W_1 and W_2 related?

Hooke's law work

$F(x) = kx$ $\quad W = \int_a^b kx\,dx$

W_1: 20 cm → 30 cm

x_i 0 $\quad x_f$ 10

$$W_1 = \int_0^{10} kx\,dx$$
$$= k\left[\frac{x^2}{2}\right]\Big|_0^{10}$$
$$= k\left(\frac{10^2}{2} - 0\right)$$
$$= 50k$$

W_2: 30 cm → 40 cm

x_i 10 $\quad x_f$ 20

$$W_2 = \int_{10}^{20} kx\,dx$$
$$= k\left[\frac{x^2}{2}\right]\Big|_{10}^{20}$$
$$= k\left(\frac{20^2}{2} - \frac{10^2}{2}\right)$$
$$= k(200-50)$$
$$= 150k$$

$$\frac{W_2}{W_1} = \frac{150k}{50k} = 3 \text{ times}$$

$$W_2 = 3W_1$$

11) a heavy rope, 50 ft long, weighs 0.5 lb/ft and hangs over the edge of a building 120 ft high.

a: How Much work is done in pulling the rope to the top of the building?

$dW = 0.5x\,dx \qquad 0 \le x \le 50$

$$W = \int_0^{50} 0.5x\,dx$$
$$= 0.5\left[\frac{x^2}{2}\right]\Big|_0^{50}$$
$$= 0.25(50^2) = 625 \text{ ft lb}$$

b: How much work is done in pulling half the rope to the top of the building?

$25 \le x \le 50$

$$W = \int_0^{25} 0.5x\,dx + \int_{25}^{50} 0.5(25)\,dx$$

$$= 0.5\left[\frac{x^2}{2}\right]\Big|_0^{25}$$
$$= 0.25(25^2)$$
$$= 0.25(625)$$
$$= 156.25$$

$$= 12.5[x]\Big|_{25}^{50}$$
$$= 12.5(50-25)$$
$$= 12.5(25)$$
$$= 312.5$$

$$W = 156.25 + 312.5$$
$$= 468.75 \text{ ft lb}$$