Determine whether the sequence converges or diverges. If it converges find the limit.

Problem 1

$$a_n = \frac{3+5n^2}{n+n^2}$$

$$\lim_{n\to\infty} \frac{3+5n^2}{n+n^2}$$

$$\frac{\frac{3}{n^2}+\frac{5n^2}{n^2}}{\frac{n}{n^2}+\frac{n^2}{n^2}} \rightarrow \frac{\frac{3}{n^2}+5}{\frac{1}{n}+1}$$

$$\lim_{n\to\infty} \frac{\cancel{\frac{3}{n^2}}+5}{\cancel{\frac{1}{n}}_0+1} = 5$$

$\boxed{\text{converges to 5}}$

Problem 2

$$a_n = \frac{3\sqrt{n}}{\sqrt{n}+2}$$

$$\lim_{n\to\infty} \frac{3\sqrt{n}}{\sqrt{n}+2}$$

$$\frac{3\sqrt{n}\,|\sqrt{n}}{\frac{\sqrt{n}}{\sqrt{n}}+\frac{2}{\sqrt{n}}} \rightarrow \frac{3}{1+\frac{2}{\sqrt{n}}}$$

$$\lim_{n\to\infty} \frac{3}{1+\cancel{\frac{2}{\sqrt{n}}}^{\,0}} = 3$$

$\boxed{\text{converges to 3}}$

8.1 Victoria Higareda

Section 8.1

AARON BALLARD

6) $\left(1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81} \dots\right)$

$$a_n = \left(\frac{1}{3}\right)^{n-1}$$

$$\boxed{a_n = \frac{1}{3^{n-1}}}$$

19) $a_n = \sqrt{\dfrac{1+4n^2}{1+n^2}}$

$$a_n = \sqrt{\dfrac{\frac{1}{n^2}+4}{\frac{1}{n^2}+1}}$$

$$a_n = \sqrt{\frac{4}{1}}$$

$$a_n = \sqrt{4} = \boxed{2}$$

$$\boxed{\text{converges to } 2}$$

31) $a_n = \left(1 + \frac{2}{n}\right)^n$

$$\left(1+\frac{x}{n}\right)^n \to e^x$$

$$\boxed{\text{converges to } e^2}$$

Section 8.2 Solutions: Question #1 Kamaran Williams

1) a) $a_n = \frac{2n}{3n+1}$ determine if sequence converges.

Step #1: Write sequence as a limit.

$$\lim_{n\to\infty} a_n \longrightarrow \lim_{n\to\infty} \frac{2n}{3n+1}$$

Step #2: Find limit by dividing by the largest n^{th} term.

$$\lim_{n\to\infty} \frac{\frac{2\cancel{n}}{\cancel{n}}}{\frac{3\cancel{n}}{\cancel{n}} + \frac{1}{n}}$$ Note: $\frac{1}{n}$ goes to zero

$$\lim_{n\to\infty} \frac{2}{3+0} \longrightarrow \lim_{n\to\infty} \frac{2}{3}$$

Answer: The sequence converges at $\frac{2}{3}$

b) Determine if series converges.

Step #1: Write sequence as a series.

$$\sum_{n=1}^{\infty} a_n \longrightarrow \sum_{n=1}^{\infty} \frac{2n}{3n+1}$$

Step #2: Test for divergence using "n^{th} term test".

$$\lim_{n\to\infty} \frac{2n}{3n+1} \longrightarrow \lim_{n\to\infty} \frac{2}{3+0} = \frac{2}{3} \neq 0$$

Note: If taking the limit of a_n as $n\to\infty$ does not equal zero then the series diverges. The n^{th} term test can not be used to determine convergence, only divergence.

Answer: The series Diverges.

Section 8.2 Solutions: Question #2 Kamaran Williams

2) Determine if series converges, if so determine sum.

$$\sum_{n=1}^{\infty} 3^{n+1} 4^{-n}$$

Step #1: Rewrite series so exponent of n is positive.

$$\sum_{n=1}^{\infty} \frac{3^{n+1} 4^{-n}}{1} \rightarrow \sum_{n=1}^{\infty} \frac{3^{n+1}}{4^{n}}$$

Step #2: Seperate exponents with dissimilar terms.

$$3^{n+1} \rightarrow 3^{n} \cdot 3^{1}$$

Step #3: Rewrite series.

$$\sum_{n=1}^{\infty} \frac{3^{1} \cdot 3^{n}}{4^{n}}$$

Step #4: Find the Ratio, something like r^n or $\frac{r^n}{r^n}$

$$\sum_{n=1}^{\infty} \frac{3 \cdot 3^{n}}{4^{n}} \rightarrow \text{ratio: } \frac{3^n}{4^n} = \left(\frac{3}{4}\right)^n$$

Step #5: Use standard geometric series formula to determine a and r.

$$\sum ar^n \qquad \sum_{n=1}^{\infty} 3\left(\frac{3}{4}\right)^n \qquad a = 3 \qquad r \text{ or ratio} = \frac{3}{4}$$

Step #6: Determine convergence or divergence using geometric series rules.

- Converges if $|r| < 1$
- diverges if $|r| \geq 1$

$r = \frac{3}{4}$ $\left|\frac{3}{4}\right| < 1$ Answer: Series Converges

Section 8.2 Solutions: Question #2 Kamaran Williams

Step # 7: Find sum of series using geometric formula.

$$\sum_{n=1}^{\infty} 3\left(\frac{3}{4}\right)^n \quad a=3 \quad r=\frac{3}{4} \qquad \text{Formula: } \frac{ar}{1-r}$$

• Plug values into formula & solve

$$\frac{ar}{1-r} \longrightarrow \frac{\frac{3}{1}\cdot\frac{3}{4}}{\frac{1}{1}-\frac{3}{4}} \longrightarrow \frac{\frac{9}{4}}{\frac{4}{4}-\frac{3}{4}} \longrightarrow \frac{\frac{9}{4}}{\frac{1}{4}} \longrightarrow \frac{9}{4}\cdot\frac{4}{1}=\frac{36}{4}=9$$

Answer: Sum of series is 9.

Final Answers for Question #2:

- Series $\sum_{n=1}^{\infty} 3^{n+1}4^{-n}$ converges.
- Sum of series is 9.

Note: Question #1 is question 15 in Section 8.2 and question #2 is question 32 in Section 8.2.

Jasmin Kisir | Section 2 | 8.2 Geometric Series

Determine whether the geometric series is convergent or divergent. If it is convergent, find the sum.

$$3 - 4 + \frac{16}{3} - \frac{64}{9} + \dots$$

$a = 3 \quad r = -\frac{4}{3} \quad |r| < 1$

$$\left|-\frac{4}{3}\right| = \frac{4}{3} > 1 \quad \boxed{\text{divergent}}$$

Determine whether the series is convergent or divergent by expressing the n^{th} partial sum S_n as a telescoping sum. If it convergent, find its sum.

$$\sum_{n=2}^{\infty} \frac{2}{n^2-1} \qquad S_n = \sum_{K=2}^{n} \left(\frac{1}{K-1} - \frac{1}{K+1}\right)$$

$$n^2 - 1 = (n-1)(n+1) \qquad S_n = 1 + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1}$$

$$\frac{2}{(n-1)(n+1)} = \frac{1}{n-1} - \frac{1}{n+1} \qquad \lim_{n\to\infty} S_n = \boxed{\frac{3}{2}}$$

Vanessa Quinones
8.2 (other than geometric)

1) $\frac{1}{3}+\frac{1}{6}+\frac{1}{9}+\frac{1}{12}+\frac{1}{15}+\dots$

$$\sum_{n=1}^{\infty} \frac{1}{3n}$$

$$= \frac{1}{3}\sum_{n=1}^{\infty} \frac{1}{n}$$

the series $\Sigma \frac{1}{3n}$ diverges by comparison to harmonic series

2) $\sum_{n=1}^{\infty} \left(\sin\frac{1}{n} - \sin\frac{1}{n+1}\right)$

$$SN = \sum_{n=1}^{N} \left(\sin\frac{1}{n} - \sin\frac{1}{n+1}\right)$$

$$SN = \left(\sin 1 - \sin\tfrac{1}{2}\right)+\left(\sin\tfrac{1}{2}-\sin\tfrac{1}{3}\right)+\left(\sin\tfrac{1}{3}-\sin\tfrac{1}{4}\right)+\dots\left(\sin\tfrac{1}{N}-\sin\tfrac{1}{N+1}\right)$$

$$\boxed{SN = \sin(1) - \sin\left(\frac{1}{N+1}\right)}$$ PARTIAL SUMS

$$\lim_{n\to\infty} SN = \sin(1) - 0$$
$$= \boxed{\sin(1)}$$ SUM OF THE SERIES

Anna Maria Hernandez
MATH 151 - 03
Direct Comparison Test - ANSWERS

Use the Comparison test to determine whether the series is convergent or divergent.

① $\sum_{n=1}^{\infty} \frac{n}{2n^3+1}$

$$\frac{n}{2n^3+1} \approx \frac{n}{2n^3} \xrightarrow{\text{behaves like}} \frac{1}{2n^2} \approx \frac{1}{n^2}$$

Compare to $\sum \frac{1}{n^2}$

P-test: $p=2>1$, convergent

Since: $2n^3+1 > 2n^3$

then: $\frac{n}{2n^3+1} < \frac{n}{2n^3} \longrightarrow \frac{n}{2n^3} = \frac{1}{2n^2}$

So:

$$0 \le \frac{n}{2n^3+1} < \frac{1}{2n^2}$$

Thus: The series $\boxed{\text{converges}}$ by direct comparison to $\sum \frac{1}{2n^2}$

② $\sum_{n=1}^{\infty} \frac{n\sin^2 n}{1+n^3}$

$0 \le \sin^2 n \le 1$

So: $n\sin^2 n \le n$

Thus: $\frac{n\sin^2 n}{1+n^3} \le \frac{n}{1+n^3}$

Since: $1+n^3 > n^3$

then: $\frac{n}{1+n^3} < \frac{n}{n^3} \longrightarrow \frac{n}{n^3} = \frac{1}{n^2}$

So: $0 \le \frac{n\sin^2 n}{1+n^3} \le \frac{1}{n^2}$

We know $\sum \frac{1}{n^2}$ converges by p-test.

↳ $p=2>1$, convergent.

Thus original series $\boxed{\text{converges}}$ by direct comparison to p-series $\sum \frac{1}{n^2}$.

Harrison De León

8.3 Limit comparison
Problems and solutions

Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{2^n-1}$ converges or diverges

$a_n = \frac{1}{2^n-1}$ $\quad b_n = \frac{1}{2^n}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{2^n-1}}{\frac{1}{2^n}} = \frac{2^n}{2^n-1} \cdot \left(\frac{1/2^n}{1/2^n}\right)$

$\lim_{n\to\infty} \frac{1}{1-1/2^n} = \boxed{1 > 0}$

Since the limit is greater than 0 and $\sum_{n=0}^{\infty} \frac{1}{2^n}$ is a convergent geometric series, $\sum_{n=1}^{\infty} \frac{1}{2^n-1}$ converges

Does $\sum_{n=1}^{\infty} \frac{2n^2+3n}{5n^5+n^2+7}$ converge or diverge

$a_n = \frac{2n^2+3n}{5n^5+n^2+7} \approx \frac{2n^2}{5n^5} = \frac{2}{5n^3}$ $\quad b_n = \frac{1}{n^3}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{2n^2+3n}{5n^5+n^2+7}}{\frac{1}{n^3}} = \frac{n^3(2n^2+3n)}{5n^5+n^2+7}$

$\lim_{n\to\infty} \frac{2n^5+3n^4}{5n^5+2n^2+7} \cdot \left(\frac{1/n^5}{1/n^5}\right) = \frac{2+3/n}{5+1/n+7/n^5}$

$\lim_{n\to\infty} \frac{2+3/n}{5+1/n+7/n^5} = \boxed{\frac{2}{5} > 0}$

Since the limit is greater than 0 and $\sum_{n=0}^{\infty} \frac{1}{n^3}$ converges, $\sum_{n=1}^{\infty} \frac{2n^2+3n}{5n^5+n^2+7}$ converges

Section 8.3 (Alternating Series)

Maria Moore.

Problem 1: Determine whether the series is convergent or divergent

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3+5n}$$

Let $b_n = \frac{1}{3+5n}$

use alternating series Test

1) $b_n > 0$ ✓

2) b_n is decreasing ✓

check limit

$$\lim_{n \to \infty} \frac{1}{\underbrace{3+5n}_{\text{goes to } \infty}} = 0 \quad ✓$$

Series converges

check absolute convergence

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^{n-1}}{3+5n} \right| = \sum_{n=1}^{\infty} \frac{1}{3+5n}$$

behaves like

$\sum \frac{1}{n}$ this diverges

therefore:

$$\boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3+5n} \text{ Converges conditionally}}$$

Problem 2: Determine whether the series is convergent or divergent

$$\sum_{n=0}^{\infty} (-1)^{n-1} \arctan(n)$$

Let $b_n = \arctan(n)$

$$\lim_{n \to \infty} \arctan n = \frac{\pi}{2}$$

limit of this series does not equal 0

n^{th} test

$$\lim_{n \to \infty} (-1)^{n-1} \arctan = DNE$$

therefore

$$\boxed{\sum_{n=0}^{\infty} (-1)^{n-1} \arctan(n) \text{ is Divergent}}$$

Alondra R.
Section 03
8.4 Alternating Series Remainder Formula

Solutions

1. We are using the first 6 terms, so we need to find the remainder after 6 terms, R_6

→ The Alternating Series Remainder Theorem says $|R_n| \leq a_{n+1}$
This means the error is no bigger than the next term after the stopping point

→ The positive part of the series is $a_n = \frac{1}{n}$

→ Since we stopped at $n=6$, the next term $= a_{6+1} = a_7$

→ Plug 7 into the formula $a_7 = \frac{1}{7}$

→ So the remainder bound is $|R_6| \leq \frac{1}{7}$

→ Rewritten as a decimal $|R_6| \leq \frac{1}{7} \approx 0.1429$
This means the approximation is within about 0.1429 of the true sum

2. The positive part of the series is $a_n = \frac{1}{n^2}$

→ If we stop after n terms, the next term will be $a_{n+1} = \frac{1}{(n+1)^2}$

→ We want the error to be less than 0.001, so we need

$\frac{1}{(n+1)^2} < 0.001$ Rewrite as → $0.001 = \frac{1}{1000}$ so → $\underbrace{\frac{1}{(n+1)^2} < \frac{1}{1000}}$

This means the denominator has to be bigger than 1,000

$(n+1)^2 > 1000$

$n+1 > \sqrt{1000}$

$n+1 > 31.62$

*n+1 must be a whole # → $n+1 > 32$ *final answer should be a whole #

Solve for n $n+1 = 32$

$n = \boxed{31 \text{ terms}}$

Check:

If you used 31 terms, the first term you leave out is term 32

$a_{32} = \frac{1}{32^2} = \frac{1}{1024} \approx 0.000977$

and since

$|R_{31}| \leq 0.000977 < 0.001$

it works

Claira Silady - Section 8.4 (Ratio Test)

1. $\sum_{n=1}^{\infty} n e^{-n} = \sum_{n=1}^{\infty} \frac{n}{e^n}$ Ratio test: $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

$L<1$ converges
$L>1$ diverges
$L=1$ inconclusive

$$L = \lim_{n\to\infty} \left|\frac{n+1}{e^{n+1}} \cdot \frac{e^n}{n}\right| = \lim_{n\to\infty} \left|\frac{n+1}{n} \cdot \frac{e^n}{e^{n+1}}\right|$$

$$= \lim_{n\to\infty} \left|\frac{n+1}{n} \cdot \frac{1}{e}\right| = \left|\frac{1}{e}\right| \lim_{n\to\infty} \frac{n+1}{n} = 1 \cdot \frac{1}{e} = \frac{1}{e}$$

$L = \frac{1}{e} = \frac{1}{2.2718} < 1$ so the series converges absolutely by ratio test

2. $\sum_{n=1}^{\infty} \frac{(-2)^n n!}{(2n)!}$

$$L = \lim_{n\to\infty} \left|\frac{(-2)^{n+1}(n+1)!}{(2(n+1))!} \cdot \frac{(2n)!}{(-2)^n n!}\right|$$

$\frac{2^{n+1}}{2^n} = 2$

$\frac{(n+1)!}{n} = (n+1)$

$$= \lim_{n\to\infty} \left|\frac{2(n+1)(2n)!}{(2n+2)(2n+1)(2n)!}\right|$$ - cancel $(2n)!$

$$= \lim_{n\to\infty} \left|\frac{2(n+1)}{2(n+1)(2n+1)}\right|$$ - cancel $2(n+1)$ $$= \lim_{n\to\infty} \left|\frac{1}{2n+1}\right| = 0$$

$0<1$ so the series converges absolutely by ratio test

RYAN WORDEN

8.4 ROOT TEST

SOLUTIONS:

Recall: Let

$$\lim_{n \to \infty} \sqrt[n]{|a_n|} = L$$

The Root Test says:

1. $L < 1 \Rightarrow$ ~~a_n is~~ series is convergent
2. $L > 1 \Rightarrow$ series is divergent
3. $L = 1 \Rightarrow$ Root Test is inconclusive

1. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$

$$\lim_{n \to \infty} \sqrt[n]{\left|\frac{n^2+1}{2n^2+1}\right|^n}$$

$$= \lim_{n \to \infty} \left|\frac{n^2+1}{2n^2+1}\right| = \frac{1}{2}$$

$\frac{1}{2} < 1$, the~~s~~ series is convergent/converges by the root test

2. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{-2n}{n+1}\right)^{5n}$

$$\lim_{n \to \infty} \sqrt[n]{\left|\frac{-2n}{n+1}\right|^{5n}}$$

$$= \lim_{n \to \infty} \left|\frac{-2n}{n+1}\right|^5$$

$$= \lim_{n \to \infty} \left(\frac{2n}{n+1}\right)^5$$

$$= 2^5$$

$2^5 > 1$, the series diverges by the root test

3. $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2}$

$$\lim_{n \to \infty} \sqrt[n]{\left|1 + \frac{1}{n}\right|^{n^2}}$$

$$= \lim_{n \to \infty} \left|1 + \frac{1}{n}\right|^n$$

$$= \infty$$

$\infty > 1$, the series diverges by the root test

Bella Esquer
Section 3
RoC / IoC

Review Answers
Lesson 8.5: Radius & Interval of Convergence

(1) $\sum_{n=1}^{\infty} 2^n n^2 x^n$

1) Ratio Test

$$\lim_{n\to\infty}\left|\frac{2^{n+1}(n+1)^2 x^{n+1}}{2^n n^2 x^n}\right|$$

$$\lim_{n\to\infty}\left|\frac{2^{n+1}}{2^n}\cdot\frac{(n+1)^2}{n^2}\cdot\frac{x^{n+1}}{x^n}\right|$$

$$\lim_{n\to\infty}\left|2\cdot\left(\frac{n+1}{n}\right)^2\cdot x\right|$$

* As $n\to\infty$, the fraction $\frac{n+1}{1}\to 1$ *
* So the limit is: $|2x|$ *

2) Find the Radius (R)

$|2x|<1 \Rightarrow |x|<\frac{1}{2}$

$\boxed{R=\frac{1}{2}}$ • making the range $(-\frac{1}{2},\frac{1}{2})$ for now...

3) Test endpoints for the interval (I)

$x=-\frac{1}{2}$

$\Rightarrow \sum 2^n n^2 x^2$

$\sum 2^n n^2 (-\frac{1}{2})^2$

$\sum 2^n n^2 \frac{(-1)^n}{2^n}$

$\sum (-1)^n n^2$

• The terms of n^2 don't approach 0 as $n\to\infty$
• By 'Test for divergence' $x=-\frac{1}{2}$ diverges

$x=\frac{1}{2}$

$\Rightarrow \sum 2^n n^2 x^n$

$\sum 2^n n^2 (\frac{1}{2})^n$

$\sum 2^n n^2 \frac{1}{2^n}$

$\sum n^2$

• The terms of n^2 grow to ∞ as $n\to\infty$
• Thus, $x=\frac{1}{2}$ diverges.

4) Concluding

$\boxed{RoC=\frac{1}{2}}$ $\boxed{IoC:(-\frac{1}{2},\frac{1}{2})}$

(2) $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(x-5)^n}{n5^n}$

1) Ratio Test

$$\lim_{n\to\infty}\left|\frac{(x-5)^{n+1}}{(n+1)5^{n+1}}\cdot\frac{n5^n}{(x-5)^n}\right|$$

$$\lim_{n\to\infty}\left|\frac{(x-5)^{n+1}}{(x-5)^n}\cdot\frac{5^n}{5^{n+1}}\cdot\frac{n}{n+1}\right|$$

$$=\left|\frac{x-5}{5}\right|\cdot\lim_{n\to\infty}\left(\frac{n}{n+1}\right)$$

$$=\frac{|x-5|}{5}$$

2) Find the Radius (R)

$\frac{|x-5|}{5}<1 \Rightarrow |x-5|<5$

$\boxed{R=5}$ • making the range $(0,10)$ for now...

3) Test endpoints for the interval (I)

$x=0$

$\Rightarrow \sum_{n=1}^{\infty}\frac{(-1)^{n-1}(-5)^n}{n5^n}$

$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(-1)^n\cdot 5^n}{n5^n}$

$\sum\frac{(-1)^{2n-1}}{n}$

$\sum\frac{-1}{n}$

• Series turned into a harmonic series
• Thus, series diverges

$x=10$

$\Rightarrow \sum\frac{(-1)^{n-1}(5)^n}{n5^n}$

$\sum\frac{(-1)^{n-1}}{n}$

• Series is an Alternating Harmonic series
- which converges by AST

4) Concluding

$\boxed{RoC=5}$ $\boxed{IoC:(0,10]}$

Solutions

Libby Poggi (8.7) McLaurin Series to find Limits/Derivatives

① $\lim_{x \to 0} \dfrac{\sin x - x + \frac{x^3}{6}}{x^5} \xrightarrow{\text{apply}} \sin x = x - \dfrac{x^3}{6} + \dfrac{x^5}{120} \cdots$

substitute: $\sin x - x + \dfrac{x^3}{6} = \left(x - \dfrac{x^3}{6} + \dfrac{x^5}{120}\right) - x + \dfrac{x^3}{6} = \dfrac{x^5}{120} + \dots$

$$\frac{\frac{x^5}{120}}{x^5} \longrightarrow \frac{1}{120} = \boxed{\frac{1}{120}}$$

② $\int \arctan(x^2)\,dx \xrightarrow{\text{apply}} \arctan(x) = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{2n+1} = u - \dfrac{u^3}{3} + \dfrac{u^5}{5} \dots$

sub in $(x^2) = \sum_{n=0}^{\infty} (-1)^n \dfrac{(x^2)^{2n+1}}{2n+1} = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{4n+2}}{2n+1} = x^2 - \dfrac{x^6}{3} + \dfrac{x^{10}}{5} - \dots$

Integrate! $\longrightarrow \int \arctan(x^2)\,dx = \int \left(\sum_{n=0}^{\infty} (-1)^n \dfrac{x^{4n+2}}{2n+1} \right) dx$ → just x term

$$\boxed{\int \arctan(x^2)\,dx = C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+3}}{(2n+1)(4n+3)} \overset{\text{or}}{=} C + \frac{x^3}{1 \cdot 3} - \frac{x^7}{3 \cdot 7} + \frac{x^{11}}{5 \cdot 11} \dots}$$

8.7

Maeve Farrell Eulers Theorem

$e^{\pi i/6}$

1) $e^{i\theta} = \cos(\theta) + i\sin(\theta)$

$\theta = \frac{\pi}{6}$

$e^{\pi i/6} = \underbrace{\cos(\pi/6)}_{=\frac{\sqrt{3}}{2}} + i\underbrace{\sin(\pi/6)}_{=\frac{1}{2}}$

$e^{\pi i/6} = \frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}$

$e^{\pi i/6} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

2) $e^{i\theta} = \cos(\theta) + i\sin(\theta)$

$\theta = 3x$

$e^{i(3x)} = \cos(3x) + i\sin(3x)$

$e^{ix} = \cos(x) + i\sin(x)$

$\cos(3x) + i\sin(3x) = (\cos(x) + i\sin(x))^3$

$= \binom{3}{0}\cos^3 x + \binom{3}{1}\cos^2 x(i\sin x) + \binom{3}{2}\cos x(i\sin x)^2 + \binom{3}{3}(i\sin x)^3$

$= \cos^3 x + 3\cos^2 x(i\sin x) + 3\cos x(i^2\sin^2 x) + (i^3\sin^3 x)$

$i^2 = -1$
$i^3 = -i$

$= \cos^3 x + 3i\cos^2 x\sin x + 3\cos x(-1)\sin^2 x + (-i\sin^3 x)$

$= \cos^3 x + 3i\cos^2 x\sin x - 3\cos x\sin^2 x - i\sin^3 x$

$= \underbrace{(\cos^3 x - 3\sin^2 x\cos x)}_{\text{real}} + \underbrace{i(3\cos^2 x\sin x - \sin^3 x)}_{\text{imaginary}}$

$\cos(3x) + i\sin(3x) = (\cos^3 x - 3\sin^2 x\cos x) + i(3\cos^2 x\sin x - \sin^3 x)$

$\cos(3x) = \cos^3(x) - 3\sin^2(x)\cos(x)$ ✓

$\sin(3x) = 3\sin(x)\cos^2(x) - \sin^3 x$ ✓

~~[illegible]~~

8.4 - Abs/Conditional Convergence ans.

- Marshall Graham

1. $\left|\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}\right| = \sum_{n=1}^{\infty} \frac{1}{n^2}$, $P = 2 > 1$, converges abs.

2. $\sum_{n=1}^{\infty} \left|(-1)^n \cdot \frac{n}{n^2+1}\right| = \sum_{n=1}^{\infty} \frac{n}{n^2+1}$

$\frac{n}{n^2} = \frac{1}{n}$

Use limit comp. to $\sum_{n=1}^{\infty} \frac{1}{n}$

$\lim_{n\to\infty} \frac{n}{n^2+1} \cdot \frac{n}{1} = \frac{n^2}{n^2+1} = 1$, diverges, not abs.

Check for conditional convergence. Using AST

$\lim_{n\to\infty} \frac{n}{n^2+1} = 0$

decreasing ✓

$\Rightarrow \sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$ is Conditionally Convergent.

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8.7 #2 Find the McLaurin Series of

$$f(x) = \frac{Ln(1+x^2)}{x^2}$$

$$Ln(1+x) \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^n}{n}$$

$$Ln(1+x^2) \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n}}{n}$$

$$\frac{1}{x^2} Ln(1+x^2) \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n-2}}{n}$$

Ratio test

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+2} x^{2n}}{(n+1)} \cdot \frac{n}{(-1)^{n+1} x^{2n-2}} \right|$$

$$\left|\frac{1}{x^2}\right| \lim_{n\to\infty} \left| \frac{(-1)(n)}{(n+1)} \right| = 1$$

$$\left|\frac{1}{x^2}\right| < 1$$

$$|x| < 1 \quad ROC = 1$$

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8.7 #1: $x^4\arctan\left(\frac{x^2}{2}\right)$ find the Mclaurin Series

$$\arctan x \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)} \quad ROC = 1$$

$$\arctan x^2 \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{4n+2}}{(2n+1)}$$

$$\arctan\left(\frac{x^2}{2}\right) \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{4n+2}}{(2n+1)\, 2^{n+1}}$$

$$x^4\arctan\left(\frac{x^2}{2}\right) \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{4n+6}}{(2n+1)\, 2^{n+1}}$$

Ratio test:

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+2} x^{4n+10}}{(2n+3)\, 2^{n+2}} \cdot \frac{(2n+1)\, 2^{n+1}}{(-1)^{n+1} x^{4n+6}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(-1)(x^4)(2n+1)}{(2)(2n+3)} \right| \Rightarrow \left| \frac{x^4}{2} \right| \lim_{n\to\infty} \left| \frac{(-1)(2n+1)}{(2n+3)} \right| = 1$$

$$\left|\frac{x^4}{2}\right| < 1$$

$$|x^4| < 2 \quad ROC = \sqrt[4]{2}$$

$$|x| < \sqrt[4]{2}$$