Math 320 Linear Algebra
Assignment # 12

1. Consider
$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 4 \\ 0 & a & -1 \end{bmatrix}$$
Given that $\det(A) = \frac{2}{3}$, find A (i.e. find a the only missing part of A).

2. An $n \times n$ matrix, A, is said to be orthogonal if $A^T A = I_n$. Suppose that A is orthogonal, show that $\det(A)$ is either 1 or -1.

3. Let:
$$A = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{bmatrix}.$$
Suppose $B = \frac{1}{3}A$.
   (a) Show that B is orthogonal.
   (b) Use this to find the possible values for $\det(A)$. (You don't need to find which one it is but you can for practice if you want).

4. Let $A = \left[\begin{smallmatrix} 2 & 3 \\ -1 & 2 \end{smallmatrix}\right]$ and $B = \left[\begin{smallmatrix} -3 & 1 \\ -2 & -2 \end{smallmatrix}\right]$. Find the following:
   (a) $\det(A)$
   (b) $\det(B)$
   (c) AB
   (d) $\det(AB)$
   (e) Show $\det(A)\det(B) = \det(AB)$.

5. Let:
$$A = \begin{bmatrix} -7 & 5 & 0 \\ -10 & 8 & 0 \\ -5 & 5 & -2 \end{bmatrix}.$$
   (a) Show that $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ is an eigenvector for A and find its corresponding eigenvalue.
   (b) Show that $\lambda = -2$ is an eigenvalue and find two linearly independent eigenvectors associated with λ.