

Math 320 Linear Algebra
Assignment # 12 Due: Tuesday, December 1

Textbook Problems:

4.1: 1, 2, 4, 10, 14, 15, 20

4.2: 2, 4, 8, 14, 20

Additional Problems:

1. Consider

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 4 \\ 0 & a & -1 \end{bmatrix}$$

Given that $\det(A) = \frac{2}{3}$, find A (i.e. find a the only missing part of A).

2. An $n \times n$ matrix, A , is said to be orthogonal if $A^T A = I_n$. Suppose that A is orthogonal, show that $\det(A)$ is either 1 or -1 .

3. Let:

$$A = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{bmatrix}.$$

Suppose $B = \frac{1}{3}A$.

- (a) Show that B is orthogonal.
 - (b) Use this to find the possible values for $\det(A)$. (You don't need to find which one it is but you can for practice if you want).
4. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 1 \\ -2 & -2 \end{bmatrix}$. Find the following:
- (a) $\det(A)$
 - (b) $\det(B)$
 - (c) AB
 - (d) $\det(AB)$
 - (e) Show $\det(A) \det(B) = \det(AB)$.

5. Let:

$$A = \begin{bmatrix} -7 & 5 & 0 \\ -10 & 8 & 0 \\ -5 & 5 & -2 \end{bmatrix}.$$

- (a) Show that $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ is an eigenvector for A and find its corresponding eigenvalue.
- (b) Show that $\lambda = -2$ is an eigenvalue and find two linearly independent eigenvectors associated with λ .