

Math 320 Linear Algebra
Assignment # 4 Due: Tuesday, October 6

Textbook Problems:

- 1.7: 34, 38
1.8: 2, 4, 8, 12, 17, 20, 32
1.9: 2, 4, 6, 10

Additional Problems:

1. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be defined by:

$$T \left(\begin{bmatrix} a \\ b \\ c \end{bmatrix} \right) = \begin{bmatrix} a + b \\ 3c \end{bmatrix}$$

- (a) Find:

$$5 \cdot T \left(\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \right)$$

- (b) Find:

$$T \left(5 \cdot \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \right)$$

- (c) Are the previous two parts equal? Does that guarantee that T is linear?
(d) Prove that T is linear by using the one step verification.

2. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be defined by:

$$T \left(\begin{bmatrix} a \\ b \end{bmatrix} \right) = \begin{bmatrix} a - 2b \\ 3b \\ 2a - b \end{bmatrix}$$

- (a) Either prove that T is linear by finding A such that $T(\vec{v}) = A\vec{v}$, or show that T is not linear with a specific example.
(b) Find $\vec{v}$ such that $T(\vec{v}) = \begin{bmatrix} 10 \\ -9 \\ 11 \end{bmatrix}$, or explain why no such $\vec{v}$ exists.
(c) Find $\vec{v}$ such that $T(\vec{v}) = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, or explain why no such $\vec{v}$ exists.

3. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be defined by:

$$T \left(\begin{bmatrix} a \\ b \end{bmatrix} \right) = \begin{bmatrix} a \\ a^2 + 1 \\ -b \end{bmatrix}$$

- (a) Either prove that T is linear by finding A such that $T(\vec{v}) = A\vec{v}$, or show that T is not linear with a specific example.
(b) Find $\vec{v}$ such that $T(\vec{v}) = \begin{bmatrix} 10 \\ -9 \\ 11 \end{bmatrix}$, or explain why no such $\vec{v}$ exists.
(c) Find $\vec{v}$ such that $T(\vec{v}) = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, or explain why no such $\vec{v}$ exists.

4. Suppose that $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ has the one-step verification property, that is for all $\vec{u}, \vec{v} \in \mathbb{R}^n$ and $c, d \in \mathbb{R}$, $T(c\vec{u} + d\vec{v}) = cT(\vec{u}) + dT(\vec{v})$. Show that $T(c\vec{u}) = cT(\vec{u})$. This finishes the proof of the one-step verification.