Math 320 Linear Algebra
Assignment # 11

1. Let $T : P_2 \to \mathbb{R}^{2\times 2}$ be defined by:

$$T(ax^2 + bx + c) = \begin{bmatrix} a+b & a \\ b & c \end{bmatrix}$$

   You can assume without proof that T is linear. (Hint: Only the first problem should require much in the way of work, the rest can be solved from theorems in the class.)

   (a) Find a basis for $\ker(T)$.

   (b) Find $\dim(\ker(T))$.

   (c) Find $\dim(\mathrm{Rg}(T))$

   (d) Find a basis for $\mathrm{Rg}(T)$

   (e) Is T 1-1?

   (f) Is T onto?

2. Let V and W be vector spaces with $\dim(V) = n$ and $\dim(W) = m$. Also $T : V \to W$ is a linear transformation. Determine which of the following statements are **always** true explain your answer.

   (a) If $m < n$ then T is not 1-1.

   (b) If $m > n$ then T is not 1-1.

   (c) If $m < n$ then T is not onto.

   (d) If $m > n$ then T is not onto.

3. Suppose that $\lambda \in \mathbb{R}$ is an eigenvalue for $A \in \mathbb{R}^{n\times n}$. (That is there exists $\vec{v_0} \neq 0$ called an eigenvector such that $A\vec{v_0} = \lambda\vec{v_0}$). Let $E_\lambda = \{\vec{v} \in \mathbb{R}^n : A\vec{v} = \lambda v\}$. Show that E_λ is a subspace of $\mathbb{R}^n$.