Math 361: Real Analysis 2
Assignment # 11

Remember you may use anything we proved in class or previous homework assignments, previous problems in the same homework assignment and even previous parts from the same question even if you did not complete them. The one exception is that if I am asking you to work on part of a proof of a theorem from class you may not use that theorem in your proof (but you may use any part we proved before the part you are proving).

You may also use the major parts of Real I. If you have a question about that please ask.

1. Remember in class we defined $a^b = exp(b\ln(a))$ when $a > 0$ and showed it is consistent with the definitions of a^q when $q \in \mathbb{Q}$.

   For each of the following $a > 0$,$b, c \in \mathbb{R}$:

   (a) Show $\exp(x) = e^x$ for all $x \in \mathbb{R}$. (Remember $e = \exp(1)$.)

   (b) Show $a^{b+c} = a^b a^c$.

   (c) Show $a^{-b} = \frac{1}{a^b}$

   (d) Show $\ln(a^b) = b\ln(a)$.

   (e) Show $(a^b)^c = a^{bc}$.

   (f) Show $(ab)^c = a^c b^c$, if $b > 0$.

   (g) Let $f(x) = a^x$ show $f'(x) = a^x \ln(a)$

2. Remember in a previous homework we defined:

$$I_n = \frac{q^{2n}}{n!} \int_0^a x^n (a-x)^n g(x)\, dx$$

   Where $q > 0$, $a > 0$ and $g \in c([0, a])$ with $g(0) = g(a) = 0$ and $0 < g(x) \leq M$ on $(0, a)$. (Note for this you don't need the conditions that $g''(x) = -g(x)$ on $[0, a]$.)

   Use previous homework results plus what we recently did in class to show that (for any choices q, a, M, g that satisfy the above conditions):

$$\lim_{n\to\infty} I_n = 0.$$

3. (a) Find $\int_1^3 x^2 \exp(x)$.

   (b) Find $\int_1^3 \ln(x)$.