Math 361: Real Analysis 2
Assignment # 14

Remember you may use anything we proved in class or previous homework assignments, previous problems in the same homework assignment and even previous parts from the same question even if you did not complete them. The one exception is that if I am asking you to work on part of a proof of a theorem from class you may not use that theorem in your proof (but you may use any part we proved before the part you are proving).

You may also use the major parts of Real I. If you have a question about that please ask.

1. Show that if $\{f_n\}$ converges to f uniformly on $A \subseteq \mathbb{R}$ then it converges pointwise on A.
2. Let $A \subseteq \mathbb{R}$ and $\{f_n\}$ be a sequence of functions are uniformly continuous on A. Suppose $f \in b((A)$ and $f_n \to_u f$. Show f is uniformly continuous on A.
3. Show that it $f_n \to_u f$ on $A \subseteq \mathbb{R}$ then $\{f_n\}$ is Cauchy with respect to the uniform norm.
4. Let $A \subseteq \mathbb{R}$. Remember we say that $\mathcal{A} \subseteq b(A)$ is complete with respect to the uniform norm if every sequence of functions $\{f_n\} \subseteq \mathcal{A}$ that is Cauchy with respect to the uniform norm then there exists $f \in \mathcal{A}$ such that $f_n \to_u f$. Determine (with proof) if the following sets are complete with respect to the uniform norm:
   (a) $c([a, b])$ where $a, b \in \mathbb{R}$.
   (b) $\mathcal{R}([a, b])$ where $a, b \in \mathbb{R}$.
   (c) $p([-M, M])$, where $M \in \mathbb{R}$ and $p(A)$ is the set of all polynomials on A.